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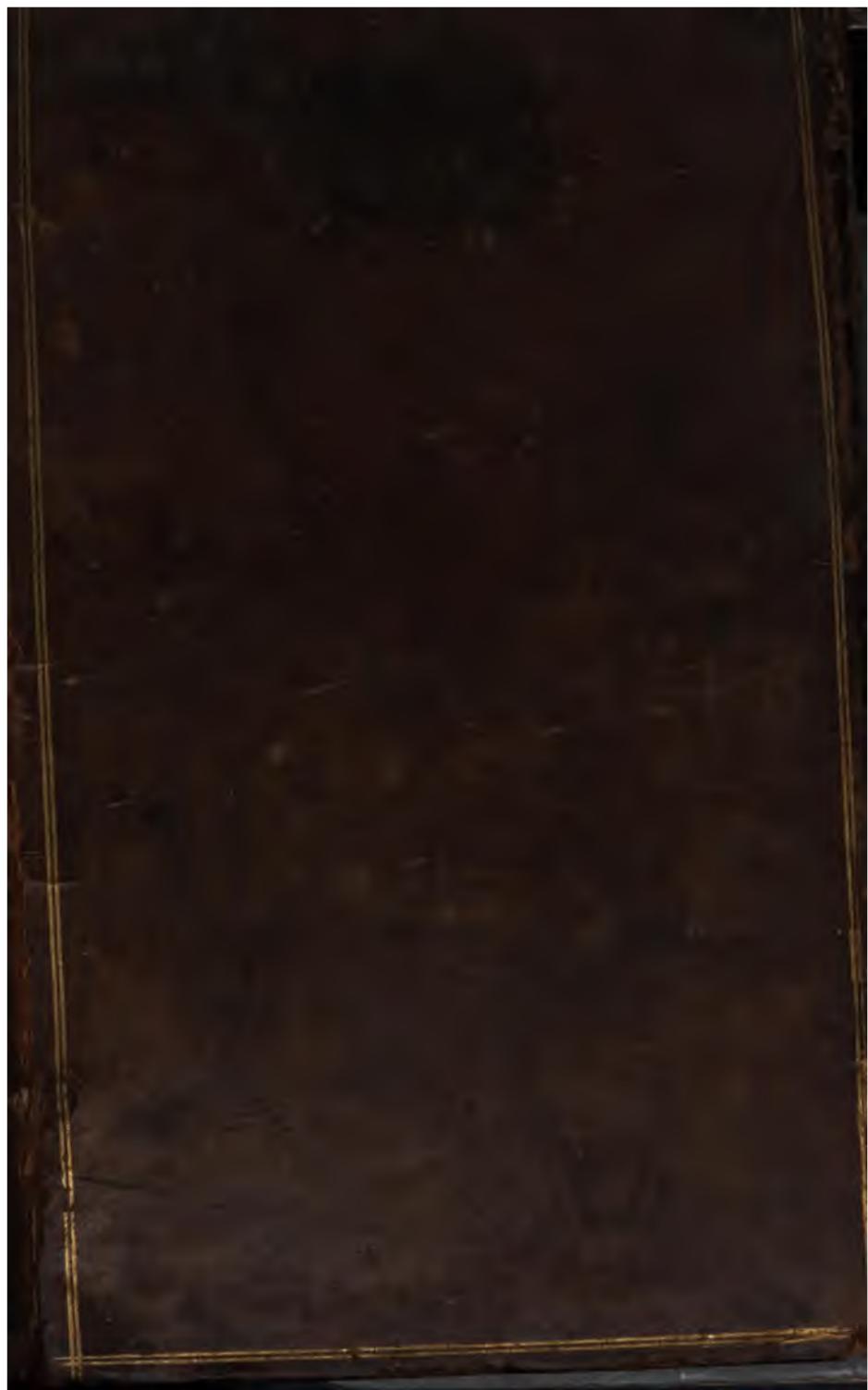
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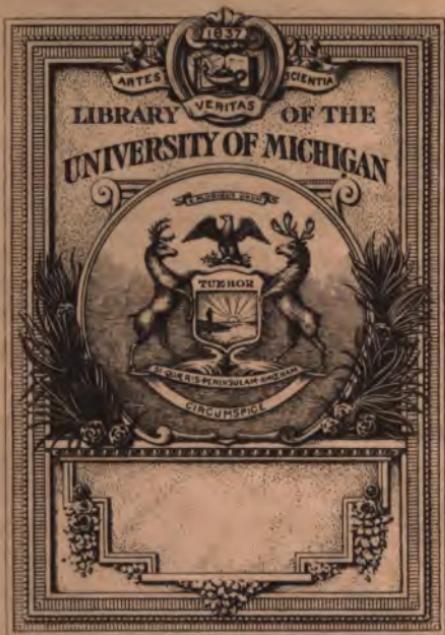
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William Padmore

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THE  
RUDIMENTS  
OF  
NAVIGATION.



THE  
R U D I M E N T S  
O F  
N A V I G A T I O N.

Demonstrated and Illustrated in a Plain and  
Familiar Manner,

By a V A R I E T Y of E X A M P L E S.

Together with the  
C O N S T R U C T I O N and U S E

o f

The Table of Logarithms.

The Lines on the Plain and GUNTER's Scales.

The Table of Natural and Artificial Sines, &c.

The Table of Meridional Parts.

The Table of Difference of Latitude, and Departure.

Compiled for the Use of the Young Gentlemen on  
board his Majesty's Ship M A G N A N I M E.

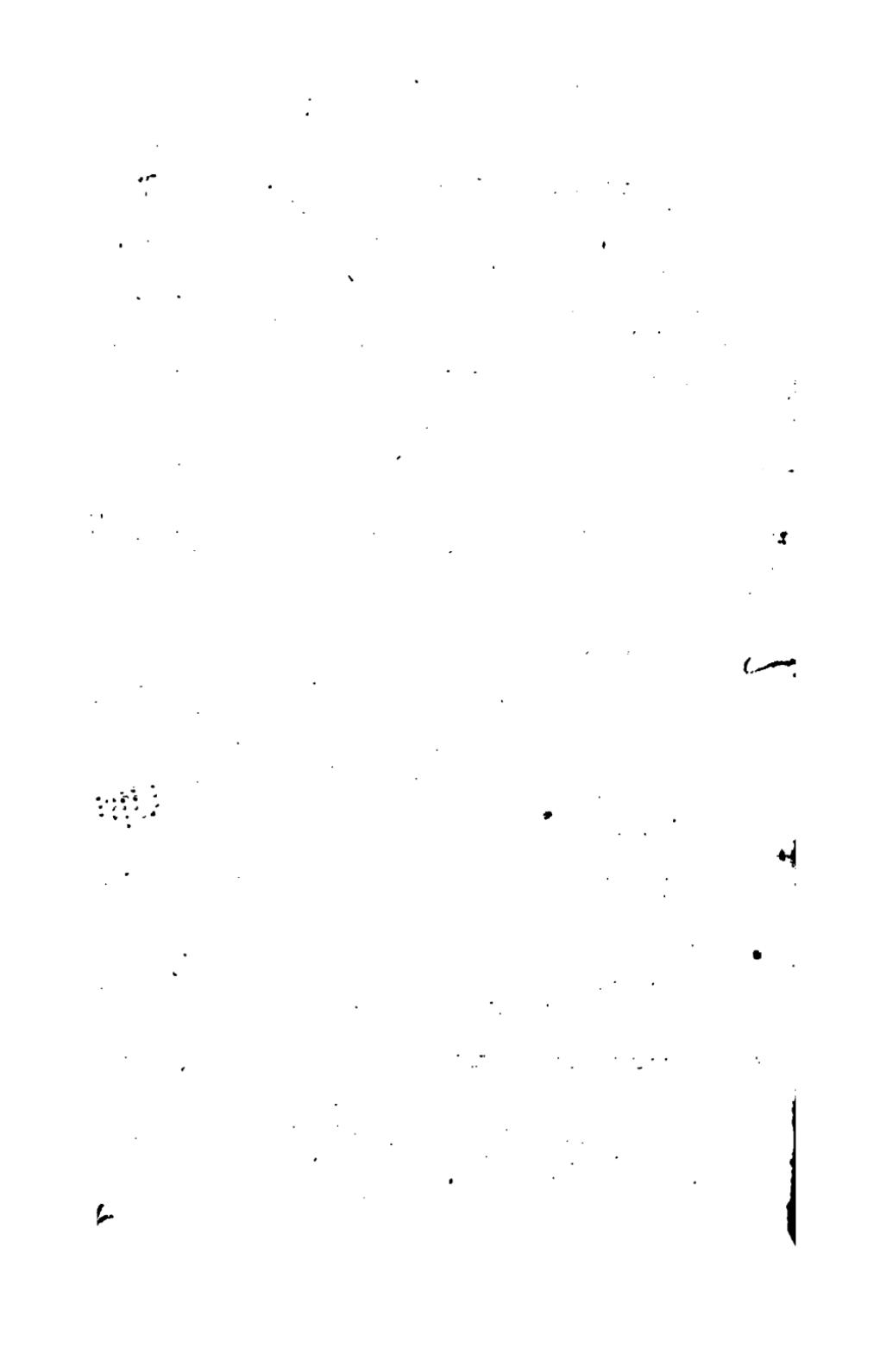
By M U N G O M U R R A Y,  
School-master of the said Ship.

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L O N D O N :

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M.DCC.LX.



T O

The RIGHT HONOURABLE  
H E N R Y,  
Earl of GAINSBOROUGH,  
Viscount CAMPDEN, &c. &c.

MY LORD,

HAVING had the Honour to instruct your Lordship in the Principles of Navigation, according to the Plan exhibited in these Sheets, I humbly conceive there is none to whom I can with so much Propriety address them as to yourself. Permit me therefore to lay them at your Lordship's Feet; and to acknowledge, in this pub-

lick

lick Manner, the grateful Sense I shall  
always retain of your Lordship's great  
Condescension, and singular Favours,  
conferred upon,

My L O R D,

*Your LORDSHIP's most Obliged,*

*And most Devoted,*

*Humble Servant,*

*Depiford,  
May, 1760.*

MUNGO MURRAY.

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## CHAP.

terms antecedent, consequent, and ratio, are explained: here we have shewn that the equality of ratios is the essential property of proportion, and that when four numbers are in a geometrical proportion, the product of the extremes is equal to that of the means: hence, the reason of the common operations in the rule of three, by multiplication and division, becomes obvious. But when the numbers consist of more than three or four places of figures, those operations will be tedious; to remedy which, we have, in the third section, taught how they may be performed very expeditiously by the logarithms, and illustrated so much of the nature of these numbers, as is necessary for understanding the principles upon which they are calculated: And, in the fourth section, we have given the construction of *Gunter's* lines, with their use in the solution of questions, in multiplication, division, and the rule of three.

The elements of geometry are the subject of the second chapter in four sections: the first contains the definitions; the second, the theoretical part of geometry; and principally, that altho' all the angles of any plain triangle make 180 degrees, yet the chords, or any other lines which measure those degrees, are greater or less in different circles, in proportion to the radius of each; and that in similar triangles, the sides about the equal angles are proportionals: So that if in two similar triangles all the sides and angles of one of them be known, and either one side and the angles, or two sides and one angle of any other triangle be also known, the unknown parts of this other triangle may be found by the rule of three, which comprehends the whole doctrine of trigonometry. The third section comprises the practical part of geometry, particularly, the construction and use of the line of chords. The fourth section treats of sines, tangents, and secants,

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must find some method of describing the circles of the sphere *in plano*: this is called the projection of the sphere, which we have exhibited in the second section, whereby the latitudes and longitudes of places are determined by plain right-angled triangles.

As all the meridians intersect one another in the poles upon the globe, they will meet in one point, when projected on any plane, as in land-maps; and of consequence, any strait line will cross them all, at unequal angles: but as the path a ship describes on the sea, by the direction of the compass, makes equal angles with all the meridians, there must be some expedient found to represent them by parallel lines.

We have therefore, in the fifth Chapter, shewn how to construct the sea-charts, in five sections. In the first, the construction and use of the plain chart is explained, wherein the meridians being represented by parallel lines, the rhumbs, which are expressed by strait lines, will cross the meridians at equal angles, and consequently, the distance, difference of latitude, and departure, always constitute a right-angled triangle on this chart; but it is evident, this must be very erroneous: for, on this projection, a degree in any parallel of latitude is equal to one on the equator. In the second section are demonstrated the principles of *Mercator's* chart, and shewn, the method of calculating Mr *Wright's* table of meridional parts, and constructing the chart by them. The third section contains the construction of all the various cases of plain and *Mercator's* sailing, upon *Mercator's* chart. The fourth shews the manner of calculating the table of difference of latitude and departure, and of working by it a day's work from the log; this being what is aimed at by the whole art of navigation, all the preceding parts

parts being only preparatory hereunto. And the fifth treats of parallel and middle latitude sailing.

Tho' the nature and reason of the several particulars taken notice of in this introduction, are fully handled, and clearly demonstrated, in the following sheets, yet, as the methods used for obtaining the necessary *data* are very uncertain, the compass itself, by which the course is directed, being subject to variation, it is no wonder if the mariner often falls into very great errors, especially with regard to the longitude; and indeed if the latitude could not be obtained by observation, navigation would become very precarious.

In the 6th Chapter we have therefore shewn how to find the latitude and variation of the compass, by celestial observation, all performed by right angled triangles, both geometrically and by calculation, with some remarks on the usual manner of observing by *Davis's* and *Hadley's* quadrant, and how to allow for the height of the eye above the horizon.

I would not be understood, by what I have here published, to disapprove of the method used by those who teach navigation on shore: I only think it will not be so convenient for those who are to be instructed at sea, because their duty on deck, in learning to work a ship, and clear her for action, besides the many interruptions they meet with from bad weather, takes up so much of their time, that they cannot have leisure for writing a course of navigation; whereas, when they have the rules printed, they need only work all the examples, according to the directions in the book, and write such remarks as shall be necessary for the perfect understanding of such parts as may seem to them obscure; and if the examples in the book are not sufficient for that purpose, as many more may be added, as shall enable

able them to give the solutions without the book, in the perusal of which, the following method may be observed.

As practical geometry consists chiefly in raising and letting fall perpendiculars, after being well acquainted with the method of performing this, they may next learn how to divide the circumference of a circle into degrees, and thereby make and measure angles ; afterwards they may proceed to construct the line of chords, which they will easily discover performs the office of the divided circle : and the examples they have in raising and letting fall perpendiculars, making and measuring angles, are such as will naturally lead them to construct all the cases of right-angled triangles. They come next to the construction of sines, tangents, and secants ; they have here likewise examples, wherein the length of the radius, and quantity of the arch, are given, thereby to find the length of the sine, tangent, and secant, in performing which they again construct all the cases of right-angled triangles : so when they come in the next place to what is properly called Trigonometry, they will perceive, it is only giving lines a different name, and the process will in all the cases be the same as before.

The next thing they come to is, the solution of all the preceding examples by calculation : in order to which, after they can clearly prove, that the sines, tangents, and secants of the degrees of different circles are in proportion to the radius of each, they are then to describe a circle, whose radius may be 100 or 1000 equal parts, and construct sines, tangents, and secants, to every five degrees in the quadrant, each of which is to be measured by the same scale, and their lengths set down in a table, regularly ruled for that purpose : afterwards they are to work all the cases in trigonometry, by multiplication

cation and division, and likewise by the logarithms ; and as the artificial sines, &c. are only the logarithms of the natural, they will easily perceive, that there is no occasion at all for the natural sines, in computing right-angled triangles.

In the projection of the sphere I have a globe properly dissected, which renders the demonstrations intelligible to ordinary capacities ; and when they have thereby gained a clear idea of the method of laying down places, by their latitudes and longitudes, both on the plane of the meridian and of the equator, they will easily understand the method and necessity of enlarging the degrees of the meridian, on *Mercator's* chart, which they are to construct from the parallel of 50 to that of 61 degrees of latitude, having previously calculated the miles in each degree of the meridian, by Mr *Wright's* method, both by the co-sines and by the secants ; and when the chart is constructed, they project upon it all the cases of plain and *Mercator's*, and the substance of parallel and traverse sailing.

The table of difference of latitude and departure is what they are next to understand, and therefore they are to make one page of it, from the artificial and natural sines : they are then to work all the preceding examples by this table, and are to compare the results with the projections, and likewise with the operations, by *Gunter's* lines ; after which they are to work the several days works from the log every day, and from thence transfer them to the journal ; and when they are capable of taking the sun's meridian altitude, they are then instructed thereby to find the latitude, and likewise the variation of the compass, both by amplitudes and azimuths, which they will perceive is done by projecting the sphere on the plane of the meridian.

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As this small treatise is printed from a manuscript I compiled for the use of some young gentlemen of distinction in the royal navy, whom I had the honour to instruct in the theory of navigation, and, I hope, not without the desired success; I have been forced to depart from the common methods, for the reasons already advanced, far from imagining, that all the authors who have wrote before me on this subject have mistaken their course: for we find, amongst them, many remarkably skilled in all the branches of the mathematics, and from whose writings I have collected the substance of this work. But I flatter myself, these Rudiments may be of some service to those who have not abilities or time to peruse those copious and prolix treatises; and that, when thoroughly understood, they may, by their own reading, acquire the whole art of navigation in its utmost extent. How far I may have answered this purpose, I leave to the candid determination of the public.

---

*Explication of the Signs.*

Plus, or more, — Minus, or less,  $\times$  Multiplied by,  $\div$  Divided by,  $=$  Equal to, : The sign of continued proportion, :: The sign of discontinued proportion.

$24+8-10\times6\div4=33$ ; that is, to 24 add 8, and subtract 10 from the sum; then multiply the remainder by 6, and divide the product by 4; the quotient will be equal to 33.

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*Of the Principles of ARITHMETIC.*

**A**RITHMETICK and GEOMETRY may be considered as the foundation of the several branches of the Mathematics; it is by them the theoretical part is demonstrated, the practical part reduced to order, and all the various solutions to the most intricate cases obtained.

As we are now going to treat of Navigation, which is, perhaps, one of the most useful branches of the mathematics, it may be expected that we should explain the principles of the two forementioned sciences; but as we presume those who intend to learn navigation, are previously acquainted with the fundamental rules of arithmetic, we shall pass over addition and subtraction, and only make such remarks on multiplication and division as shall be necessary to give our readers a distinct idea of the doctrine of proportion.

SECTION I.

REMARKS on Multiplication and Division.

*Rem. I.]* In multiplication there are three terms, that is, two factors and a product; one of the factors is called the multiplicand, and the other the multiplier.

multiplier ; these two being known, the third, or the product, is found by adding the multiplicand to itself, or repeating it as many times as there are units in the multiplier.

*Rem. II.]* In division, there are likewise three terms ; the divisor, dividend, and quotient. The first two being known, it is the business of this rule to find how many times, or parts of a time, the divisor is contained in the dividend ; the answer to which is the quotient.

*Rem. III.]* The greater the multiplier is, the greater is the product ; and the greater the divisor, the less the quotient : So if the multiplier or divisor be 1 or an unit, the product will be equal to the multiplicand ; and the quotient equal to the dividend ; but if the multiplier be a fraction, the product will be less than the multiplicand, and if the divisor be a fraction, the quotient will be greater than the dividend.

*Rem. IV.]* Multiplication is the reverse of division, and the contrary ; for if any product be made a dividend, and one of the factors a divisor, the other factor will become the quotient, and if any quotient be multiplied by the divisor, the dividend will become the product.

It is not our intent here to treat professedly of fractions, we shall only observe that a fraction is a part of any thing ; thus if two loaves were to be equally divided amongst three people, it is plain this could not be done without cutting each loaf into three equal parts, and then each person would have two thirds of a loaf expressed thus  $\frac{2}{3}$ , that below the line is called the denominator, expressing the number of parts the thing is divided into ; and that above it is called the numerator, signifying the number of these parts that are expressed by the fraction.

*Rem. V.]* When the multiplier is a fraction, and the

The multiplicand multiplied by the numerator, then if the product be divided by the denominator, the quotient will be the required product: Thus,

$$\begin{array}{r}
 32 \quad 32 \quad 32 \quad 32 \quad 32 \quad 32 \quad 32 \\
 \hline
 8 \quad 4 \quad 2 \quad 1 \quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8} \\
 \hline
 256 \quad 128 \quad 64 \quad 32 \quad 16 \quad 8 \quad 24 \\
 \hline
 4) 96(24
 \end{array}$$

In every one of which the multiplicands are the same; and the multipliers, in the first six, are each one half of the next before it, and therefore, by the preceding remarks the products will be each the half of the next preceding. Now when the multiplier is 1, the product is 32, therefore when the multiplier is  $\frac{1}{2}$  or  $\frac{1}{4}$ , 32 must be divided by 2 or 4, but in the last case the multiplier is treble the preceding one, and of consequence the product must be treble the preceding product, as by the operation.

As we shall have occasion, in the following Sections, to refer to these Remarks, we shall here subjoin some Examples, to illustrate what has been said on that head.

*Example 1.* How many minutes in 29 degrees, supposing 60 to a degree?

$$\begin{array}{r}
 29 \\
 60 \\
 \hline
 \end{array}$$

*Answer* 1740

*Ex. 2.* A ship 9 miles distant from the Lizard, fails 5 days on one course at the rate of 5 miles in an hour, and then meets another

Days 5  
 Hours 24  
 Hours 120  
 Miles 5  
 Miles 600

ship at sea, who wants to know her distance from the Lizard. It is admitted she had been sailing 120 hours, which multiplied by 5, makes 600, and with the 9 she was distant from the Lizard when the account began, makes 609 miles.

*Ex-*

## Of Multiplication and Division.

Ex. 3. A ship at sea in a calm tries the current, which runs at the rate of 2 miles and  $\frac{1}{2}$  in an hour, how many miles has she drove in 24 hours? Answer 60; for  $24 \times \frac{1}{2} = 12$  and  $24 \times 2 = 48$  and  $48 + 12 = 60$ .

Ex. 4. In 678 miles, how many degrees? Answer  $11^{\circ} 18'$ ;  $6,0)67,8(11^{\circ} 18'$

Ex. 5. Suppose a ship has been drove 36 miles by a current running at the rate of  $\frac{1}{2}$  a mile per hour, how many hours has she been driving? Here it is plain the divisor being a fraction, the quotient will be greater than the dividend, so the answer is 72;  $\frac{1}{2})36(72$ , for 72 halves make 36.

Ex. 6. Let 10000 be a multiplier, the multiplicand unknown, but the product is 6170880 required the multiplicand? Here, by remark the 3d, 617,0880 is the multiplicand, for unit neither multiplies nor divides.

Ex. 7. Let 960 be a multiplicand, the multiplier unknown, but we know that if 10000 were multiplied by 617, it would give a product equal to 960 multiplied by the unknown number, which is the multiplier now required, Answer  $6427\frac{8}{96}$ .

for  $617 \times 10000 \div 960 = 6427\frac{8}{96}33$ .

Rem. 6.] When several numbers are to be multiplied or divided by one another, it is indifferent in what order the operations are performed, for if 12 be multiplied by 5, that product divided by 4, and this quotient multiplied by 6, and this last product divided by 3, we shall find this last quotient to be 30 by different operations, for 4 and 3 are divisors, and 5 and 6 multipliers in both operations.

$$\begin{array}{r}
 12 \\
 \times 5 \\
 \hline
 4) 60 (15 \\
 \hline
 6 \\
 \hline
 3) 90 (30
 \end{array}$$

$$\begin{array}{r}
 4) 12 (3 \\
 \hline
 6 \\
 \hline
 3) 18 (6 \\
 \hline
 5 \\
 \hline
 30
 \end{array}$$

## Of Proportion.

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### SECTION II.

The whole body of mathematics is chiefly concerned in comparing quantities with one another; their mutual relation is called proportion. All quantities may be represented either by numbers or by lines, the operations in the first are performed by arithmetic, and in the latter by geometry. We shall here consider proportion in respect of numbers; in treating of which we shall use the following signs:  $=$  equal to;  $\times$  multiplied by,  $\div$  divided by,  $+$  more or added to;  $-$  less or subtracted from: So  $12 \times 5 \div 4 \times 6 \div 3 = 30$  signifies 12 multiplied by 5, divided by 4, multiplied by 6, divided by 3, is equal to 30; and  $6 + 8 - 5 = 9$ , signifies 6 more, 8 less, 5 equal to 9. *See the end of the introduction.*

Mathematicians explain geometrical proportion by the terms antecedent, consequent and ratio. When two things of the same kind are compared, the first is called the antecedent, the second the consequent, and when the second is divided by the first, the quotient is called the ratio: As supposing the things compared were miles; 2 the antecedent, and 6 the consequent, then will 3 be the ratio; but if 6 be the antecedent and 2 the consequent, then  $\frac{1}{3}$  will be the ratio. There can be no comparison or ratio between things of different kinds, as betwixt miles and hours, for 6 hours cannot be divided by 2 miles.

*Inference.* If any two of the three terms antecedent, consequent, and ratio, be known, the third may be found by multiplication or division; if the two first be given, as 2 the antecedent, and 6 the consequent; or 6 the antecedent and 2 the consequent; the ratio is found by dividing the consequent by the antecedent; so  $6 \div 2 = 3$  the ratio; and  $2 \div 6 = \frac{1}{3}$  the ratio: If the antecedent and ratio be given, as 2 the antecedent and 3 the ratio  $2 \times 3 = 6$

is the consequent, or if 6 be the antecedent and  $\frac{1}{3}$  the ratio, then  $6 \times \frac{1}{3} = 2$  will be the consequent, by the 5th remark in the preceding section. If the consequent and ratio be given, and the consequent divided by the ratio, the quotient will be the antecedent. Let the terms be as before: Then  $6 \div \frac{1}{3} = 2$  the antecedent, and  $2 \div \frac{1}{3} = 6$  the antecedent. When three numbers are compared, if the ratio of the first to the second, be equal to that of the second to the third; as  $2:6:18$ , they are said to be in a continued geometric proportion, and the terms separated by two points as above, 2 is to 6, as 6 is to 18, for the ratio of 2 to 6 is  $\frac{1}{3}$  equal to that of 6 to 18: and  $18:6:2$ , are likewise in a continued proportion, the ratio being  $\frac{1}{3}$ , so 18 is to 6, ~~and~~ 6 is to 2.

When four numbers are compared, and the ratio of the first to the second equal to that of the third to the fourth, but unequal to that of the second to the third; as  $2:6::5:15$  or  $21:7::18:6$ , they are in a discontinued proportion; the ratio of 2 to 6, or of 5 to 15, is 3, but that is not the ratio of 6 to 5, therefore the second and third terms are separated by four points as above, that is to say 2 is to 6, as 5 is to 15; and 21 is to 7, as 18 is to 6; in this last the ratio is  $\frac{1}{3}$ .

It is about four numbers in a discontinued proportion that the Rule of Three is chiefly concerned, for if the first three be known, this rule directs how to find the fourth, the principles of which may be deduced from the following Theorem.

### T H E O R E M.

When four numbers are proportionals, the product of the extremes is equal to that of the means, and when three numbers are in a continued proportion, the product of the extremes is equal to the square of the middle term, that is the middle term multiplied by itself.

*Note,*

*Note.* When the numbers are properly placed they are all called terms ; the first and fourth, are extremes, second and third, the means.

In order to prove the theorem, let  $a$  be the first, and  $e$  the third term,  $r$  the ratio ; then the terms may be expressed thus,  $a$ , 1st term :  $a \times r$ , 2d term ::  $e$ , 3d term :  $e \times r$ , 4th term. It is obvious, by rem. 6, sect. 1, that  $a$ , 1st term,  $\times e \times r$  4th term =  $a \times r$  2d term  $\times e$  3d term.

Now as any numbers may be applied to these letters, this theorem will be universally true, for let  $a=4$ ;  $r=5$ ;  $e=6$  then the terms will be  $4:4 \times 5::6:6 \times 5$ , that is  $4:20::6:30$ ; but  $4 \times 5 \times 6=4 \times 6 \times 5$ , that is  $4 \times 30=120$  and  $20 \times 6=120$ .

When three terms are in a geometric proportion, the middle term may be taken twice, as  $2:6:18$  may be expressed  $2:6::6:18$  and then  $2 \times 18=36$ ; and  $6 \times 6=36$ .

Having thus proved the theorem ; we come now to the rule of three itself ; which is as follows.

### R U L E.

I. Place the numbers in such a manner, that when the fourth is found, the first may have the same proportion to the second, that the third has to the fourth.

II. Multiply the second by the third term, and divide the product by the first term, the quotient will be the number required ; that is, the fourth term : The only difficulty will be, as there are but three terms given, how to place them, which we shall illustrate by the following example.

If a man walks 13 miles in 4 hours, how many will he walk in 24 hours ? Here two of the terms are hours, *viz.* 4 and 24, which must therefore be the first and second terms, because there can be no ratio between things of different kinds, as was before observed, so shall 13 be the third term, which is of the

the same kind with the fourth term, they being both miles. It yet remains to know whether 4 or 24 shall be the first term; and because we know something about 4, that is he walks 13 miles in that time, therefore 4 must be the first term; so they may be placed thus,  $4: 24 :: 13$ . And  $24 \times 13 \div 4 = 78$ , the fourth term required; as to the operation it is indifferent whether 13 or 24 be the third term, but we shall in all the examples in this work, place them so that the first and second terms may be of the same name, so shall the third and fourth be likewise of the same name.

The reason of this operation appears very plain from the preceding theorem, and the remarks in multiplication and division; for though the fourth term is not known, we are sure if it were multiplied by 4, the product would be equal to  $24 \times 13 = 312$ ; now we have got a product and one factor, therefore  $312 \div 4 = 78$  must be the other factor, the fourth term required.

It will be proper to work for answers to the following questions as we shall apply the same numbers to triangles.

QUESTION I. If 10000 yards of cloth cost 6428*l.*, what will 960 yards cost?       $10000: 6428 :: 960$ .

II. A privateer cost 10000*l.* the share of prize money coming to the proprietors was 8391*l.* one of them had 735*l.* in the common stock, what must his share of the prize money be?

$10000: 8391 :: 735$ .

III. Two partners, A, and B, put 10000*l.* into a joint stock, of which A's share was 617*l.* finding this not sufficient they added 1191*l.* to the stock, what must A pay?

IV. If 960 yards of broad cloth cost 735*l.* what will 10000 yards cost?       $960: 735 :: 10000$ .

V. If

V. If 960*l.* laid out in trade in the course of 12 years gain 617*l.* what will 10000*l.* gain in the same time at that rate? 960 : 617 : : 10000 :

VI. If 617*l.* pay 735 labourers, how much will pay 10000.

735 : 617 : : 10000 :

These examples we presume sufficient to illustrate the principles of the rule of three; we shall therefore proceed, according to the plan, to shew how the operations may be shortened by the logarithms.

### SECTION III.

#### *Of the Construction and Use of the Logarithms.*

It is not our business here to calculate tables of those admirable numbers, that being already done to great exactness by several eminent mathematicians. The learned are obliged for this useful discovery to the indefatigable labour of the noble inventor Lord *Neper*; but it will be proper here to explain so much of their nature as may enable us to understand the principles upon which they have been calculated.

Logarithms are artificial numbers, adapted to natural numbers, and so contrived that the work of multiplication is, by them, performed by addition, and that of division by subtraction. From this description the following inferences may easily be deduced, *viz.*

*Inference I.* Every natural number must have a proper logarithm, and therefore we must have a table to find them by inspection; it will be sufficient for our purpose to have them to all natural numbers under 10000, which we have in most books of navigation.

*Inf. II.* If the logarithm of any number be increased, the correspondent natural number will likewise be increased, and the greater the natural number, the greater its logarithm.

C

*Inf. III.*

*Inf. III.* If the logarithm of any number be added to itself, or, which is the same thing, if it be doubled, the sum will be the logarithm of the square of the natural number, that is, when it is multiplied by itself.

*Inf. IV.* If the logarithms of any two numbers are known; the logarithm of the product or quotient of these two numbers may with certainty be found; for the sum of the two logarithms will be the logarithm of their product, and the difference of the two logarithms will be the logarithm of their quotient.

In order then to make a table of logarithms, we may assume any number at pleasure; for the logarithm of the natural number 10, which let be 1000000, it is certain, by *Inf. 3*, that the logarithm of 100 must be 2000000, the logarithm of 1000, 3000000; of 10000, 4000000: Let then 10 be multiplied by 10, its logarithm must be doubled, which will give the logarithm of 100. Again, if to the logarithm of 100 be added that of 10, we shall have the logarithm of 1000, &c. as in the following table.

Num.	Logarithms.
10	1000000
10	1000000
<hr/>	<hr/>
$10 \times 10 = 100$	2000000
<hr/>	<hr/>
10	1000000
<hr/>	<hr/>
$100 \times 10 = 1000$	3000000
<hr/>	<hr/>
10	1000000
<hr/>	<hr/>
$1000 \times 10 = 10000$	4000000

By this table it is evident that the logarithms of all numbers from 10 to 100 begin with 1; from 100 to 1000 with 2; and from 1000 to 10000 they begin

begin with 3, &c. This initial figure is called the characteristick or index of the logarithm, and denotes how many figures are in the natural number, which will always contain one figure less than there are units in the index: for this reason the index is separated from the rest by a point. The logarithms of all numbers under 10, will be less than 1000000, by *Inf.* 2, and contain but six figures; but to make them contain the same number of figures with the other logarithms, 0 is prefixed to them; so 0 is the index of all the logarithms of the numbers under 10. And because 1 or unit neither multiplies nor divides, its logarithm must neither increase nor diminish any other: So the logarithm of 1 is 000000. In order to find logarithms to all the intermediate numbers betwixt 1 and 10, 10 and 100, &c. Those who understand the extraction of the square root, may find the logarithm of 9, 7, and 2, by the following method; first take half the logarithm of 10, and 2dly add the logarithm of 10 thereto; 3dly, take half that sum, and 4thly, add the logarithm of 10 to each: Thus, first extract the square root of 10; 2dly, multiply this root by 10; 3dly, extract the square root of this product; and 4thly, multiply this last root by 10.

In this manner we may proceed by adding the logarithm of 10 to that of the root; the half of the logarithm of the sum will be the logarithm of the root of that product, till the root comes to be above 9. Now the preceding root being less than 9, we may add the logarithms of these two roots, and half that sum will be the logarithm of this product, which will be nearer 9 than any of the preceding roots; and so, by continual additions and halvings, and extracting the roots of the several products which the inventor, with great labour, has done to 7 or 8 places of decimals,

decimals, till the root at last came to 9; the logarithm of which he found to be 0.954242: After the same method we may find the logarithms of 2, 7, 11, 13, 17, 19, and, by adding or subtracting these, we may find the logarithms of all numbers under 23. The same process, may be continued till the table is compleat.

	Num.	Logarithms.
Root of 10	3.163	0500000
10 x 3.163	31.63	1500000
Root of 31.63	5.623	0750000
10 x 5.623	56.23	1750000
Root of 56.23	7.499	0875000
10 x 7.499	74.99	1875000
Root of 74.99	8.66	0937500
10 x 8.66	86.6	1937500
Root of 86.6	9.306	0968750
	8.66	0937500
8.66 x 9.306	80.59	1906250
Root of 80.59	8.977	0953125

In the above operation when the root is 9.306, to the logarithm thereof add that of 8.66, the root next less to 9, and halving the sum we have the logarithm of 8.977, which is nearer to 9; it would be needless to continue the process, our design being only to shew how they may be constructed. Now having the logarithm of 9, the half of it 0.477121 is the logarithm of 3: Again, if 0.301030 be the logarithm of two, then the double of it 0.602060 must be the logarithm of 4: And because  $10 \div 2 = 5$  if the logarithm of 2 be subtracted from that of 10, we shall have 0.698970 the logarithm of 5;  $3 \times 5 = 15$ , therefore 1.176091 will be the logarithm of 15, for that is the sum of the logarithms of 5 and of 3; and

**SECT. III.** *Construction of Logarithms.* 13

and for the same reason, 1.397940 is the logarithm of 25, for it is double the logarithm of 5: The logarithm of 7 is 0.845098, to which adding the logarithm of 3, we shall have 1.322219, the logarithm of 21; by the same method we shall find 1.544068 to be the logarithm of 35; and 1.690196 the logarithm of 49.

This we presume sufficient to shew how the table may be made, and that it will answer the end of multiplication and division, and of consequence all operations in the rule of three, which we shall illustrate by the same examples which have been done before by multiplication and division.

*Examples in Multiplication and Division.*

**Example I.**

Logarithms.		
Deg.	29	1.462398
Min.	60	1.778151
Min.	1740	3.240549

**Example II.**

Logarithms.		
Days	5	0.698970
Hours	24	1.380211
Miles	5	0.698970
Miles	600	2.778151

**Example III.**

Mil.	2 $\frac{1}{2}$ 2.5	1.397940
Hours	24.	1.380211
Miles	60.0	2.778151

**Example IV.**

Deg.	678	1.831230
Min.	60	1.778151
Deg.	11.3	1.053089

**Example V.**

A ship drives, dividend, Miles	36.0	2.556302
Current $\frac{1}{2}$ a mile, divisor,	0.5	0.698970
Hours, quotient,	72	1.857332

When fractions are to be multiplied or divided, reduce them to decimals, and work as if they were whole numbers, and the index of the result will denote how many figures will be in the product or quotient: As, in Example III, the index of the sum of

of the logarithms is 2, therefore the natural number corresponding thereto, will have three figures; but because there must be as many decimals in the product as in both the multiplier and multiplicand, one of them must be a decimal, as in the operation in Example V, 1 is the index of the remainder, or difference of the logarithms, therefore there must be two figures in the quotient, and because there must be just as many decimals in a quotient, as the dividend has more than the divisor, which in this case is none, they must be both whole numbers. We presume this method is better than perplexing ourselves with negative indexes.

*Examples in the Rule of Three.*

The 2d example, about the privateer, when done by multiplication and division would stand thus, 10000 : 8391 :: 735; now because the second must be multiplied by the third, their logarithms must be added, and because the product must be divided by the first term, its logarithm must be subtracted from the sum of the other two logarithms. The terms may be placed in different lines, and their logarithms against them as in the following operation.

As 10000	4.000000
is to 8391	3.923814
so is 735	2.866287
to 616.8	2.790101

Here the sum of the two logarithms is 6.790101, but as the logarithm of the 1st consists only of cyphers, it is only subtracting 4 from the index as above, the nearest logarithm I find in the tables, is 790144, without regarding the index, the natural number corresponding thereto is 6168; but because the index is 2, the last figure must be a decimal. After the same manner the other questions may be solved, which we shall omit; but as we have frequent occasion to mention

tion decimals, it will be proper here to say something of them.

It was before observed, that a fraction is a part of something, and that the denominator expresses the number of parts the whole thing is divided into, and the numerator, how many of those parts are contained in the fraction; let a ruler of one foot long be divided into 12 equal parts, then six of these parts, that is half a foot, will be  $\frac{6}{12}$ , which is called a vulgar fraction: But if the foot were divided into ten equal parts; the half of a foot would be five of these parts: Again  $\frac{3}{2}$  or  $\frac{1}{4}$  of a foot would be equal to two tenths and half a tenth; which is  $\frac{1}{4}$  of a foot divided into ten equal parts; and, in order to express this, every tenth part must be divided into ten equal parts, by which the foot will be divided into 100 equal parts, and half a tenth will be  $\frac{5}{100}$  and 2 tenths  $\frac{20}{100}$ , so  $\frac{3}{2}$  will be equal to  $\frac{25}{100}$  and  $\frac{9}{100} = \frac{75}{100}$ .

When the denominator of a fraction is 1 with any number of cyphers annexed, it is then called a decimal fraction: The denominator is always 1, with as many cyphers as there are figures in the numerator, for which reason there is no occasion to set down the denominator, that being known by the numerator, to which there is a point prefixed, to distinguish it from a whole number. Thus .5.25.666. signifies  $\frac{5}{10}$ ,  $\frac{25}{100}$ ,  $\frac{666}{1000}$ . The advantage of decimals is, that by them multiplication and division is performed as in whole numbers, the only difference is in the value of the product, or quotient, but as it has been noted that there are always as many decimals in the product as in both the factors together, and of consequence, just as many decimals in the quotient, as the dividend contains more than the divisor, the value of each may easily be found. From this description of vulgar

vulgar and decimal fractions, it is obvious a vulgar may be reduced to a decimal by the following rule; as the denominator of the vulgar is to its numerator, so is 10, or 100, or 1000, &c. to the numerator of the decimal; so  $\frac{6}{10} = .6$ , for  $12:6::10:5$ , and  $\frac{3}{10} = .25$ ; for  $12:3::100:25$ , and  $\frac{8}{100} = .666$  nearly,  $12:8::1000:666$ , which may be produced to as many places as you please, and though it will never be exactly equal to the vulgar fraction, yet the difference is so very small that  $\frac{8}{100}$  of a foot or 8 inches may be reckoned equal to .666 of a foot.

S E C T. IV. *Of Gunter's Lines.*

We have in the preceding sections shewn how to work all questions in the rule of three either by multiplication, or division, or by the logarithms; we shall now shew how they may be performed by scale and compasses; in order to which we shall first remark, that addition or subtraction may be performed by a good scale of equal parts, and a pair of compasses, which will be sufficiently illustrated by the two following examples.

Example I. Let the two numbers to be added be 301, and 477.

Draw a strait line AB, and with a pair of compasses, take 301 off the scale; which set off from A to C; take also 477 off the same scale, which set off from C to D; the whole line AD measured on the same scale will be found to be 778, the sum of the two numbers. *Plate I. Fig. I.*

Example II. Let it be required to subtract 301 from 1000.

Take 1000 off the scale which lay upon the line from A to B; take also 301 off the same scale, which lay off from B to E, so shall AE, measured on the same scale, be 699, which is the remainder. *Plate I. Fig. I.*

Now,

Now, if any two numbers may be thus added or subtracted, we may do the same by the logarithms of two numbers; the two precedings are the logarithms of 2 and of 3, and of consequence their sum is the logarithm of 6. Hence it is plain multiplication and division, and of consequence all the operations in the rule of three, may be performed by compasses, and a good scale of equal parts, provided we have a table of logarithms to find the numbers that are to be added or subtracted. Mr *Gunter* has constructed lines, on which the logarithms may be had without the table, which are upon the scales called by his name, *Gunter's scales*; one of them is a line of numbers, marked 1, 2, 3, &c. to 9; and the same figures on the line, produced, at the same distance as in the first part of the line, so there is 1 at the beginning, 1 at the middle, and 1 at the end of the line.

Mr *Gunter* constructed this line from the logarithms, and because 0 is the logarithm of 1, the figure 1 is placed at the beginning of the line. Again, as 301030 is the logarithm of 2, it must be laid off from 1 to the figure 2; in like manner, the figures 3, 4, &c. are placed on the line, by setting off the logarithms of these numbers from 1 at the beginning of the line; and as 1000000 is the logarithm of 10, 1 is again placed in the middle of the line; and because  $1:2::10:20$ ,  $1:3::10:30$ ; the figures 2, 3, 4, &c. are placed at the same distance from 1 in the middle, that they are from 1 at the beginning. The lines of equal parts, on the plain scale, are only divided into 100 equal parts, which we may call 1000, and take that for the logarithm of 10, and so the logarithm of 2 will be 301. We shall shew, in the next chapter, how to make scales of 10000 equal parts, which being constructed, we may then take the logarithms by

any of these scales, as suppose by the line A B, *Pl. I.* *Fig. I.* which is a line of equal parts, and from thence transfer them to the line *G P*, which will be a line of numbers properly graduated, and is evidently a line of logarithms ; in like manner the intermediate divisions may be taken from the logarithms.

The line being thus constructed, our next business must be to read it, or find any number, or rather the logarithm of any number upon the line. Now the first figures may be reckoned 1, 2, 3, that is one, two, three, &c. then the same figures on the second part of the line, will be 10, 20, 30, &c. if the figures in the first part be accounted 10, 20, 30, &c. they will, in the second part, be 100, 200, 300, &c. The value of the figures being determined, will also determine the value of the divisions betwixt them, for the spaces betwixt them are divided into ten unequal parts ; when figure 1, at the beginning, is one, the intermediate divisions, in the first part, will be tenths of an unit, and, in the second part, they will be units ; if the first figure 1 be ten, the intermediates, in the first part, will be units, and in the second part, tens : where the spaces will admit, these ten divisions, which are distinguished by longer strokes, are divided into five or into two unequal divisions, by smaller strokes ; the short strokes, when they are four, will be two tenths each ; but when only one, it will be only one half of the adjoining long strokes. It will now be easy to find any number upon the line ; for, if it were required to find 7, and 14 ; let the first figure 1 be one ; the figure 7 next to it will be seven ; figure 1 in the middle will be ten, and counting four of the long strokes, beyond figure 1 in the middle, we have 14. Again, let two numbers be 79 and 129 ; the first 1 must be accounted ten, then counting nine strokes beyond figure 7, we have 79 ; now 1 in the middle

middle will be 100, the long strokes betwixt 1 and 2 will be tens, and the short ones two units, each so counting two of the long strokes, we have 120, and four short ones make 128; we must then estimate the half of the distance betwixt the last short stroke and the figure 2, which point will be 129.

The lines being thus constructed, we have no more occasion for the table of logarithms, for we have them upon the line, which therefore may be added or subtracted by a pair of compasses, and of consequence all questions in the rule of three, may be solved by this line; observing the following general rule.

### R U L E.

Place the numbers as before directed, and extend the compasses from the first to the second term; then placing one foot of the compasses, in the third term, the other foot with the same extent, will reach to the fourth term.

### E X A M P L E.

Let the given terms be  $2 : 8 :: 5$ , the extent from 2 to 8, will reach from 5 to 20, which is the fourth term, here we add the difference betwixt the logarithms of 2 and of 8 to the logarithm of 5, evidently same as if the logarithms of 8 and of 5 were added, and the logarithm of 2 subtracted from their sum, which is done when the operation is performed by the pen. As to multiplication and division, if 1 be put for the first term, it will be as 1 is to the multiplier, so is the multiplicand to the product; and if 1 be put for the second term, it will be as the divisor is to 1, so is the dividend to the quotient, which is in effect the rule of three; so if it were required to multiply 8 by 5, it would be  $1 : 8 :: 5 : 40$ , and the extent from 1 to 8 would reach from 5 to 40. Again, if it were required to divide 40 by 2, it would be  $2 : 1 :: 40 : 20$ , and the extent from 2 to 1 will reach from 40 to 20. We

We might add more examples, but to make us thoroughly acquainted with this line, it will be proper to work several examples with the pen, and prove them by this line; and when the product does not exceed three figures, the results will agree.

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## C H A P. II.

## Of GEOMETRY.

WE have in the preceding chapter explained the doctrine of geometric proportion by numbers; we come now to shew how this may be done by means of lines, without any arithmetical calculation.

SECTION I. *Geometrical Definitions.*

1. A definition is the perfect explanation of any word or thing supposed not to be understood.
2. A circle is a figure or space contained within one curve line, and in practice is described by a pair of compasses.
3. The center is the point where one foot is fixed, the other being carried about describes the curve.
4. The circumference or periphery is the curve line described by the moving point, as A B D E, *Pl. I. Fig. 2.*
5. The radius or semi-diameter, is any strait line drawn from the center to the circumference, as C A, C B, C D, C E, of which there may be any number, but they will be all equal to one another.
6. Diameter is any strait line drawn within the circle

## Sect. I. Geometrical Definitions.

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circle through the center, from one side of the circle to the opposite, as A D, and E B.

7. Chord is any strait line drawn from one part of the circumference to the other, so it does not pass through the center as B R, R T, T E.

8. Arch is any part of the circumference.

9. A degree is the 360th part of the circumference, for every circle may be conceived to be divided into 360 equal parts, as in the figure, the inner contains as many parts as the outer one does; but the degrees are greater or less in proportion to the radiusses of the circles.

A semi-circle is half a circle, and contains 180 degrees; a quadrant is  $\frac{1}{4}$  of a circle, that is 90 degrees; what an arch wants of 90 degrees is called the complement of that arch;  $40^\circ$  is the complement of  $50^\circ$ , and  $50$  the complement of  $40^\circ$ . The supplement of an arch is what it wants of 180 degrees, 140 is the supplement of  $40^\circ$ .

11. When two lines meet at one point they form what is called an angle, the lines that form it are called the sides or legs, the point where they meet is called the angular point. *Pl. I. Fig. 3.*

The quantity of an angle is not estimated by the length of the sides that form it, but by the number of degrees it contains, for let there be two angles A B C, and D E F, and one arch described from B, and another from E as centers, and likewise the two radiusses equal; if then the two arches be equal the angles will likewise be equal, otherwise not. We use three letters to express an angle, the middle one denoting the angular point.

12. A perpendicular is a strait line, so drawn to another, as to incline to neither side, as D C to A B, and if it is not at the end of a line, will make two equal angles, and therefore 90 degrees

grees each ; they are called right angles. When the line is not perpendicular as C F, it will incline toward A, and the angles will become unequal, and are called oblique ; the angle A C F which is less than a right one, is acute ; the angle F C B greater than a right one is called obtuse. *Pl. I. Fig. 4.*

13. Parallel lines are such as are every where equally distant from one another, as A B, and F G. *Pl. I. Fig. 1.*

14. A plain rectilineal triangle is a figure, or a space, upon a plane limited by three strait lines, which are called sides, and by their meeting form three angles ; if the sides be equal it is called equilateral : If one of the angles be right, that is 90 deg. it is called a right angled triangle. As it is only of these we shall treat, when we mention the word triangle, it must be understood of a strait lined right angled one, unless the contrary be expressed. The sides of a right angled triangle are distinguished by different names, which we shall call their proper names ; for we shall hereafter have occasion to give them sir-names.

15. Hypotenuse, is the side opposite to the right angle, which we shall call A C. The two sides that form the right angle, are called the one the perpendicular, and the other the base indifferently, but for distinction we shall call A B the perpendicular, and draw it parallel to the margin of the book ; B C we shall call the base, and draw it parallel to the top and bottom of the page, so the angle at B will be 90 deg. the angles at A and C both together will be 90 deg. as shall be proved hereafter, the angle at A we shall call simply the angle, which we shall always suppose opposite to the base. The angle at C we shall call the complement

angle

angle. The following are such natural inferences from the definitions, that they require no demonstration. *Pl. I. Fig. 5.*

## INFERENCES.

1. The greatest angle that can be made by two strait lines will be less than 180 degrees.

2. If a right line C F, meets another line A B any where as at C, so as to form two angles, both together will make 180 degrees: For, if from the angular point C, a semi-circle be described, it will exactly measure both the angles. All the angles that can be made at a point on one side of a line will make 180 degrees; but if they cross in one point, they will make 360 degrees. *Pl. I. Fig. 4.*

3. If there are several equal chords, as E F, F G, in the same circle, the arches subtended by them will also be equal, but if the circles be unequal, the arches may contain the same number of degrees in each, but the chords will be unequal in proportion to the radiusses; that is, if the radius C E be double or triple the radius C N; then will the chord E F, be double or triple the chord N M: Though this may seem plain enough by the figure, yet as upon it the whole of trigonometry chiefly depends, it shall be demonstrated in Theorem VI. It is likewise very plain, that though the arch E F G be equal to the two arches E F, and F G, taken together, yet the chord G E is not equal to the chords E F, and F G. *Pl. I. Fig. 2.*

## SECT. II. GEOMETRICAL THEOREMS.

## THEOREM I.

If two strait lines A B and D E cross one another in C, the opposite angles A C D, and B C E, will be equal; for let the angle D C B, be any number

number of degrees suppose 100; then the angles A C D, and B C E, must each be 80 degrees, by Inference 2d, Sect. 1. Chap. 2. *Pl. I. Fig. 6.*

### T H E O R E M II.

If a strait line E F cut two parallel lines A B and C D, then

1. The external opposite angles A G E and D H F will be equal; as will also the angles B G E and C H F; for the two lines being parallel, they may be considered as one broad line, and the line E F crossing it, so the angles will be equal by the preceding. *Pl. I. Fig. 7.*

2. The internal angle C H E, is equal to the external one A G E, and E H D equal to E G B, for A G E was just now proved equal to D H F, equal C H E by preceding; and for the same reason  $E G B = C H F$   $E H D$ . *Pl. I. Fig. 7.*

*Note.* By external is understood out-side; and by internal in-side.

3. If a line C B be drawn to meet two parallels A B and C D, the alternate angles A B C and B C D will be equal, for producing A B to E; D C to F; and C B to H and G; then the angle H B E is equal to the angle A B C, and likewise to the angle F C G  $= B C D$ , therefore A B C  $= B C D$ , and for the same reason the alternate angles E B C and B C F are equal. *Plate II. Fig. 12.*

### T H E O R E M III.

If a perpendicular be drawn from C, the center of a circle, to the chord A B, it will bisect the chord in F, and the arch in D; and if a perpendicular be erected from the middle of a chord, it will pass through the center, and therefore if produced will be a diameter: for the dotted lines C A and C B, being equal, also D A and D B, the point C

the

is equally distant from the points A and B, and for the same reason, the point D is equally distant from the points A and B, so F, or any other point in the line C D, will be equally distant from the points A and B, and therefore A B is bisected in F. *Pl. I. Fig. 8.*

#### T H E O R E M IV.

If a strait line A B be drawn to touch the circumference in the point B, and the chord B F be drawn, the angle A B F made by the chord and tangent, is measured by half the arch, of which B F is the chord.

#### D E M O N S T R A T I O N.

Draw the radius C E perpendicular to the chord, which will bisect the arch in E, so shall B E be half the arch F E B, evidently the measure of the angle B C E ; all then that is to be proved, is, that the angles A B F and E C B are equal : In order to which draw the radius C B, which will be perpendicular to the tangent, draw also the diameter D G parallel to the chord B F. *Plate I. Fig. 9.*

The angles A B F and F B C, both together, make 90 degrees ; the angles E C B and B C G are likewise 90 degrees ; but F B C and B C G are alternates to the parallels F B and D G, and therefore equal (by the 2d The.) and of consequence the angles A B F and B C E must be equal.

To illustrate this by numbers, suppose the arch B E F 80 degrees, the angle B C E must be 40 degrees, and B C G 50 : and because the angles F B C and B C G are equal, F B C must be 50 likewise, and of consequence the angle A B F must be 40 equal to B C E, the thing that was to be proved.

## THEOREM V.

An angle made by two chords meeting in the circumference is measured by half the opposite arch, that is the angle L H G is measured by half the arch L E G, of which L G is the chord. *Plate I. Fig. 10.*

## DEMONSTRATION.

Draw the line K M to touch the circle in H. The three angles at H make 180 degrees (by *Inf. II. Sect. 1. Chap. II.*) the angle G H M is measured by half the arch G P H; and the angle K H L is measured by half the arch H R L (by preceding) therefore the angle L H G is measured by half the arch L E G, for the half of this arch, and the halves of the other two arches, make 180 degrees: hence we may deduce the following useful inferences.

## INFERENCES

1. All the three angles of any strait lined triangle make 180 degrees, for any triangle, as we shall shew in Problem 6, may be inscribed within a circle; then each angle will be at the circumference, and the sides will become chords of arches, the halves of which measure their opposite angles, but half the sum of these arches is 180 degrees.

This inference is of great importance in trigonometry, from whence, if one of the oblique angles of a right angled triangle be known, the other is found by subtraction; we shall therefore demonstrate it another way.

Draw the line H M through T, or any of the angular points of the triangle T S N; and let it be parallel to N S, the side opposite to the angle through which the line H M is drawn. The three angles at T make 180 degrees, but the angles M T S, and T S N being alternates are equal by Theorem II, and for the same reason the angles T N S and H T N

H T N are equal, and therefore the angles at N and S, together with the angle N T S, make 180 degrees. *Plate I. Fig. 11.*

2. The angle at the circumference of a circle is half that at the center, if both stand upon the same arch, for this is measured by the whole, and that by half the opposite arch.

3. An equilateral triangle is also equiangular, for being inscribed within a circle, the sides will become equal chords, and of consequence the arches will be equal, that is, 120 degrees each, which will make the angles 60 degrees each.

4. The radius of a circle is equal to the chord of 60 degrees of the same circle; for making an equilateral triangle on the radius A C; the angle at C will be 60 degrees, that is to say the arch B F A will be 60 degrees, of which A B is the chord, evidently equal to the radius. It is on this account we always take the chord of 60 degrees before we can make or measure an angle by a line of chords.

*Plate I. Fig. 12.*

5. In two triangles, 1st if the sides of the one be respectively equal to the sides of the other; 2d, if two sides and the included angle in both be equal; 3d, if two sides and the angle opposite to the like side be equal; 4th, if two angles and the side betwixt them be equal: in all these cases the triangles will be equal in all respects.

### T H E O R E M VI.

In any triangle A B C, if a line D E be drawn parallel to any of the sides, suppose to B C; then the side A B : A C :: A E : A D; and A C : C B :: A D : D E. Also B C : B A :: E D : E A; and C B : C A :: D E : D A. *Plate I. Fig. 13.*

### D E M O N S T R A T I O N.

Divide the line A B into any number of equal parts.

parts, suppose six, in the points 12, 24, &c. thro' these points draw lines parallel to B C, which will divide the line A C into six equal parts in the points 15, 30, &c. Again, through the points, in the line A C, draw lines parallel to A B, which will divide the line B C into six equal parts, in the points 8, 16, &c. so shall the side A C be 90, the side A B 72, and B C 48. It is evident that

$$A B : B C :: A E : E D$$

$$72 : 48 :: 60 : 40$$

$$A C : C B :: A D : D E$$

$$90 : 48 :: 75 : 40$$

$$B C : B A :: E D : E A$$

$$48 : 72 :: 40 : 60$$

$$C B : C A :: D E : D A$$

$$48 : 90 :: 40 : 75$$

now wherever the line be drawn, suppose  $r t$ , at  $5\frac{1}{2}$  in the line A B, then will A  $t$  be  $\frac{5}{7}\frac{1}{2}$  of the line A B, and it is plain A  $r$  will be  $\frac{5}{7}\frac{1}{2}$  parts of the line A C, and  $r t$   $\frac{5}{7}\frac{1}{2}$  of the line B C. *Pl. I. Fig. 13.*

#### I N F E R E N C E S.

1. The chords, sines, tangents, and secants of similar arches, are in proportion to their radiusses, for the arches, A L E, and B K H, being similar, that is to say of the same number of degrees, the chords, sines, tangents, and secants, of the one, are evidently similar to those of the other; and C E : E A :: C H : H B, &c. *Plate I. Fig. 14.*

2. In similar triangles, the sides containing the equal angles are proportionals; that is, if the angle at A be equal to that at F, the angle at B equal to that at H, then will the angle at C be equal to that at G, and because the angles at A and F are equal, if A  $t$  be made equal to F H, and A  $r$  equal to F G, then the triangle A  $r$   $t$  will be equal to the triangle F G H, and the line  $r t$  parallel to B C; so shall A B : A  $t$  = F G : : A C : C  $r$  = F H. *Pl. I. Fig. 13.*

3. If

3. If two triangles are similar, and the sides and angles of one of them be known, and one side and the angles of the other, or two sides and one angle be known, then the other two sides may be found by the rule of three, for let all the sides of the triangle A B C be given, and let the triangle F G H be similar to it; and the side H G given; then to find the sides F G and F H it will be  $B C :: C A :: G F : H G$  and  $B C : B A :: H G : H F$ .

We shall have occasion to have recourse to this inference in trigonometry, for all the parts of a triangle, similar to the triangle concerned, must be known, before we can obtain the things required.

We shall now proceed to the practical part of geometry, contained in the following problems.

### S E C T. III. Geometrical Problems.

#### P R O B L E M I.

To divide a strait line A B into two equal parts,  
*Plate I. Fig. IV.*

1. With one foot of the compasses in A, with any convenient extent describe two small arches, the one above, and the other below the line.

2. With the same extent, and one foot in B, describe other two arches to cut the former in D and E, and draw the line D E, which will cut the line A B in two equal parts in C; for it is evident the points D and E are the same distance from A that they are from B, and the line D E being the nearest that can be drawn betwixt these two points, all the parts of it will be the same distance from A, they are from B, and of consequence the point C will be in the middle of the line; in practice there will be no occasion to draw the line D E, as the point C may be found by laying a ruler from D to E.

P R O.

## P R O B L E M II.

At the point C, near the middle of a line, to erect a perpendicular.

1. With any convenient extent take two points A and B equally distant from the point C. *Pl I. Fig. 4.*

2. Describe one arch from A and another from B to intersect one another in D, so shall D C be the perpendicular required.

## P R O B L E M III.

From the point D to let fall a perpendicular to the line A B near the middle of it.

1. With any convenient extent from D, as center, describe two arches to cut the line in any two points, as in A and B. *Plate I. Fig. 4.*

2. Describe one arch from A, and another from B, with the same radius, to intersect one another in the opposite side of the line, in E; and lay a ruler from D to E, so shall D C be the perpendicular required. These two need no demonstration, for the points D and E being the same distance from A they are from B, and A and B being equally distant from C, the line D C inclines no more to one side than to the other.

## P R O B L E M IV.

From the point B at the end of the line to erect a perpendicular.

1. Assume any convenient point C out of the line, in which place one foot of the compasses, and open the other to B. *Plate I. Fig. 15.*

2. With the radius C B describe one arch to cut the line at A, and another arch opposite thereto.

3. Lay a ruler over A and C to cut the opposite arch in D, so shall D B be the perpendicular required.

## P R O B L E M V.

To let fall a perpendicular from the point D, when it will be near the end of the line.

This is only the reverse of the former: for draw a line from the point D, to intersect the line any where as in A, and find C the middle of this line, from which as center, with the radius C D = C A, describe an arch to cut the line in B, so shall D B be the perpendicular required: for it is plain if the semi-circle A B D be described from the center C, A D will be a diameter, and the angle D B A at the circumference, which will therefore be measured by half the opposite arch (by Theorem 5.) In this case 180 degrees, therefore the angle at B will be 90 degrees.

*Plate I. Fig. 15.*

## P R O B L E M VI.

To describe a circle through any three points A B D which are not situated in a strait line. *Pl. I. Fig. 16.*

1. Draw the chords A B and B D, for they will be chords, because the points must be in the circumference.

2. Bisect the chords by two perpendiculars to intersect one another in C, which will be the center required.

## P R O B L E M VII.

To draw a line parallel to a given one A B.

1. If the distance betwixt the lines be given; with that, as radius, draw one arch from A, and another from B, and draw the line C K just to touch these arches, which will be parallel to A B. *Plate II. Fig. 1.*

2. If the point N be given, through which the line is to be drawn.

*1. Fix*

1. Fix one foot of the compasses in the given point  $N$ ; and open the other to cut the given line any where suppose at  $B$ .
2. With that radius, and one foot any where in the line  $A B$ , as at  $M$ , describe an arch.
3. With  $M B$  as radius, from the center  $N$  describe an arch to cut the former in  $K$ , so shall  $K N$ , be parallel, to  $A B$ .

### DEMONSTRATION.

Draw the line  $K B$ ; so shall the triangles  $K B N$  and  $K B M$  be equal, for  $K N = B M$  and  $K M = N B$  by construction, and  $K B$  common to both, therefore the alternate angles  $K B M$ , and  $B K N$ , are equal, and the lines parallel by Theorem II.

### PROBLEM VIII.

To make an angle  $A B C$  equal to a given one  $D E F$ .

Describe an arch from the center  $E$ , and one from the center  $B$ , both with the same radius, which may be assumed at pleasure, and make this last arch equal to the former, so shall the angles be equal by Def. 13. *Plate I. Fig. 3.*

By the same method we make an angle of any number of degrees, but, as was before observed, we must first divide a circle into degrees by the following method.

1. Describe a circle and quarter it with the two diameters  $A D$  and  $E B$ , so shall each quarter be 90 degrees. *Plate I. Fig. 2.*
2. Take the radius in the compasses, which lay off on the circumference both ways from the points  $A$ ,  $B$ ,  $D$ , and  $E$ , this will divide the quadrant into three equal parts, and therefore 30 degrees each: Again, these subdivided into three equal parts, will divide the quadrant into nine equal parts, which will be

be ten degrees each, this is sufficient for our purpose, our intention being only to shew how the circumference may be divided, which the mathematical instrument makers have effected, not only to single degrees, by repeated trials, but even to minutes and seconds, by a contrivance which shall be explained in another place. After one circle is divided, any other circles may be divided by drawing them from the same center, and producing the radii of the divided circle to the circumferences that are to be divided.

*Plate I. Fig. II.*

Being now provided with a divided circle, we may make an angle of any number of degrees, which is in effect making an angle equal to a given one; for let it be required to make an angle of 40 degrees, we have the angle H C B in the quadrant A B which is 40 degrees; so there is no more to be done, but to take the radius of this, or indeed of any divided circle, and with that describe an arch, the center of which must be the point where the angle is to be made, and then take the required number of degrees from the periphery of the divided circle, and set them off on the arch. In practice this is done by a line of chords, for it must be observed, that in measuring arches we do not apply an instrument to the curve, but take the distance, betwixt the extremities of the arch, with a pair of compasses, which in effect is taking the chord of the arch. Now, if from the point B lines be drawn to the several divisions of the arch B D, they will be chords of the arches 10, 20, &c. and B D will be the chord of 90 degrees. We may then from the center B describe arches through the several divisions of the quadrant to intersect the line B D in the points 10, 20, &c. which will be evidently equal to the respective chords in the quadrant; and it is by this method the lines of chords

on the plain scale are constructed. In order to make an angle by a line of chords: first, with the chord of 60, describe an arch, this will be the same as if we take the radius of the divided circle, because the chord of 60 degrees is equal to it, by Inference 4, Theorem V. Secondly, take the required number of degrees from the line of chords, which will be the same thing as if taken from the circumference, and lay them off on this arch, as shall be illustrated in the following examples; for it is in making and measuring angles; raising and letting fall perpendiculars, that the practical part of Geometry is chiefly employed. There are several lines of equal parts on the plain scales, these parts may represent feet, yards, miles, or any other measure, and as all arches of circles, or which is the same thing, all angles, are measured by a line of chords; so are the lines that form the angles measured by a line of equal parts; great care must therefore be taken not to mistake the one for the other, as it often happens with beginners. It is presumed the following questions may be solved without any further directions than referring to the preceding problems. We shall only observe that A B is to be drawn parallel to the margin of the book, A C oblique, and B C parallel to the top or bottom. *Plate I, Fig. 5.*

QUESTION I. Let it be required to make an angle of 40 degrees at the point A of the line A B, then to make A C 960 miles, and from the point C to let fall a perpendicular to cut the line A B in B; required the miles in the lines A B and B C, and the degrees in the angles at B and at C?

II. After making the angle at A 40 degrees, as in the preceding, make the line A B 735 feet, and the angle at B 90 degrees, required the angle at C and the length in feet of the lines A C and B C?

III. Erect

III. Erect a perpendicular at the point B, on which lay off 617 leagues from B to C, and then make an angle at C of 50 degrees; how many leagues the lines A C and A B? and how many degrees the angles at A and at B?

IV. After making a right angle at B, make B A 735 feet, and A C 960 feet. Then required the angles at A and at C, and the length of the line B C?

V. Make a right angle at B, as in the preceding, then make B C 617 feet, and C A 960 feet, required the oblique angles; and the line A B?

VI. Make a right angle at B as before, make B A 735, and B C 617 feet required the line A C, and the angles at A and at C?

In the resolution of these questions, it is plain a right angled triangle has been constructed in each; the angle at B being always 90 degrees; the angles at A and C both together have likewise made 90 deg. the angle at C, as was before observed, is the complement of that at A; the line A C the hypotenuse; A B the perpendicular; and B C the base. We shall find when we come to trigonometry, that these questions contain all the varieties of right angled triangles.

VII. Let the radius of a circle be 960 feet; how many feet the chord of 40 deg. of that circle?

VIII. Let the radius of a circle be 735 feet, and a chord in that circle 503, how many degrees the arch?

### P R O B L E M IX.

To divide a strait line, A B, into any number of equal parts, suppose 10, 100, or 1000. *Plate II. Fig. 2.*

1. Divide any line C D into ten equal parts, by a good scale of equal parts, or assume any quantity for one of these parts; and set it off ten times from C to D.

2. Make

2. Make  $CD$  the side of an equilateral triangle,  $FCD$ ; and the lines  $FA$  and  $FB$  equal to  $AB$ ; so the lines  $FD$  and  $FC$  must be produced, when  $CD$  happens to be shorter than  $AB$ , which is the line to be divided.

3. Draw lines from the point  $F$ , through the several divisions of the line  $CD$ , which produced will divide the line  $AB$  into ten equal parts. Again, to divide it into 100 equal parts, each of these must be subdivided into ten equal parts; therefore take  $Ax$ , which is the tenth part of the line  $AB$ , and set it from  $F$ , to  $R$ , and  $T$ ; then draw the line  $RT$ , which will be equal to  $Ax$ , and will be divided into ten equal parts, by the lines already drawn from  $F$ ; for the triangles  $FR_1$ ,  $FCz$ , and  $FAx$  are similar; therefore  $FC : Cz :: FA : Ax$ , but  $Cz$  is the tenth part of  $FC$  or  $CD$ ; therefore  $Ax$  must be the tenth part of  $FA$  or  $AB$ , and  $R_1$  the tenth part of  $RT = Ax$ , so shall  $R_1$  be the hundredth part of the line  $AB$ ; and if this could admit of being divided into ten equal parts, the line  $AB$  would be divided into 1000 equal parts; but here the divisions would be too small, we must therefore have recourse to the following method:

1. Draw the lines  $AB$ , and  $CD$ , parallel to one another, and the lines  $AC$ , and  $BD$ , perpendicular to them.

2. Divide each of these four lines into ten equal parts; by the preceding, and draw the parallels and diagonals as in the figure. *Plate II. Fig. 3.*

Now in the triangle  $ACF$  are nine lines parallel to the base, which will constitute ten similar triangles; the first base will be one tenth, the second base two tenths, &c. of the base  $CF$ : these are called diagonals, by which one inch, half, or a quarter of

of an inch, may be divided into 100 equal parts as those upon the plain scales. *Pl. II. Fig. 3.*

It may not be improper here to remark, that as by one line of chords any circle may be divided; so likewise by one good scale of equal parts, any strait line may be divided into any proposed number of equal parts, or into unequal parts, which shall be in a given proportion to one another; as for instance, if it were required to divide the line A B into five unequal parts, which shall have the same proportion to one another as the following numbers.

2, 5, 7, 9, 4. *Plate II. Fig. 5.*

To do this, draw the line C D, which make 27 by any scale of equal parts, and lay off the several parts as in the figure; then make C D the base of an equilateral triangle F C D, and make F E and F G equal to A B, so shall E G likewise be equal to A B, and lines drawn from F to the several divisions of the line C D, will divide the line E G into the same number of parts and, in the same proportion with those of the line C D.

We may likewise, by a good scale of equal parts, measure the several parts of any strait line, provided the length of the whole line be known; as if it were required to find how many equal parts are contained betwixt the figure 1, and the figures 2, 3, 4, &c. of the line of numbers on Gunter's scales, the whole line from 1 to 1 being 1000. To perform this, *Plate II. Fig. 4.*

1. Take the whole line in the compasses, with which from the center A describe an arch.

2. Take 1000 off the half-inch diagonal scale, which lay on the arch from B to C, and draw the lines A B and A C.

3. Set off the several divisions of the line of numbers from the point A, on the lines A B and A C; the figure 1 being at the point A, the distances

tances betwixt the several figures on the line A B, and the same figures on the line A C, measured on the half-inch scale, will be the number of equal parts contained betwixt those figures and 1 at the beginning of the line: and upon examination will be found to be the logarithms of those numbers.

*Plate II. Fig. 4.*

It is on the same principles the Sector is constructed, which performs these and several other useful operations very expeditiously.

#### S E C T I O N IV.

##### Of Sines, Tangents, and Secants.

###### D E F I N I T I O N S.

1. The right sine of an arch is a perpendicular let fall from one end of the arch to the radius or diameter drawn thro' the other end of it: A D is the right sine of the arch A B, and is always half the chord of double the arch; for making E N equal to E A, it is evident the sine A F is half the chord N A. *Pl II. Fig. 6.*

2. The co-sine, or sine complement of an arch, is equal to that part of the radius intercepted betwixt the right sine and the center; C D is equal to F A the sine of the arch E A, the complement of the arch A B; and the versed sine of an arch is that part of the radius intercepted betwixt the right sine and the circumference; D B is the versed sine of the arch A B; so the co-sine and versed sine both together make the radius.

3. A Tangent to a circle, is a strait line so drawn as to touch a circle only in one point, and if to this point be drawn a radius, it will be perpendicular thereto, because it is the shorlest that can be drawn from the center to the circumference; so B M is a tangent to the circle in the point B. *Pl. II. Fig. 6.* The

The tangent of any arch, as of  $AB$ , is that part of the line  $BM$ , intercepted betwixt the point  $B$ , at one end of the arch, and the point  $H$ , where a line drawn from the center, thro' the point  $A$  at the other end of the arch, meets the line  $BM$ ; so  $BH$  is the tangent of the arch  $AB$ , and  $EG$  the co-tangent, or tangent complement, that is the tangent of the arch  $EA$ . *Plate II. Fig. 6.*

4. The secant of an arch, as of  $AB$ , is a line drawn from the center, thro'  $A$  at one end of the arch, and produced till it meet a perpendicular erected from  $B$  at the other end of the arch; so  $CH$  is the secant of the arch  $AB$ , and  $CG$  the co-secant or secant-complement of the same arch. *Fig. 6.*

The sine, tangent, and secant of an arch, are the same with those of the supplement; for being drawn according to the preceding definitions, the same lines will result.

*Inferences from these Definitions.*

1. The sine of 90 degrees is equal to the radius, for it is half the diameter or chord of 180 degrees; the sine of 30 degrees is half the radius; or which is the same thing, half the chord of 60 degrees.

2. The tangent of 45 degrees is equal to the radius; for let the arch  $KS$  be equal to the arch  $SE$ , they will be 45 degrees each;  $RK$  is the tangent of the arch  $SK$ , and is parallel and equal to  $EC$  the radius; but the radius cannot be equal to any secant. *Plate II. Fig. 6.*

3. The radius, sine, and co-sine of any arch constitute a right angled triangle. In the triangle  $ACD$ , right angled at  $D$ , the radius  $CA$  is the hypotenuse, the sine  $AD$  is the perpendicular, and the sine complement  $CD$  is the base. The radius, tangent, and secant of any arch do also make a right angled triangle. In the triangle  $CBH$  right angled.

angled at B, the radius CB is the base, the tangent BH the perpendicular, and the secant CH the hypotenuse. *Plate II. Fig. 6.*

4. The length of the sines, tangents, and secants, of any arch, cannot be determined, till the length of the radius be fixed, even tho' the number of degrees contained in the arch be known; but having the length of the radius given, suppose CB, then the sines, tangents, and secants to every degree, and even to every minute, may be drawn, and their lengths measured; in the figure it is done to every ten degrees, which is sufficient for our purpose, where it is evident, the tangents and secants of the degrees of the quadrant of the small circle, are just half the tangents and secants of those of the great circle, the radius of the one being double that of the other; and it is by this means that the lines on the plain scale are constructed; first actually drawing them to the several divisions of the arch, and then transferring them from thence to the scale, where there is a line of equal parts adapted to the radius, which contains 1000 of these equal parts. Now when the several sines, tangents, and secants are measured on this scale, we may make a table by which the sines, tangents, and secants of every degree may be found by inspection to a radius of 1000 equal parts. *Plate III. Fig. 1 and 2.*

Having thus explained the construction of the sines, tangents, and secants, we come now to shew that all the varieties of right angled triangles may by them be solved.

It was before observed, that the sides of a right angled triangle were distinguished by different names, *viz.* hypotenuse, base, and perpendicular; these we shall call their proper names. It is also evident they may likewise be considered as sines, tangents, or secants, of the arches that measure the oblique angles. Hence

Hence the whole business of trigonometry consists in finding sines, tangents, or secants; whether the quantity of the arch, and the length of the radius, or the length of the sine, tangent, or secant of the arch be known; or if these be known, to find the quantity of the arch: and although all the varieties may be reduced to two cases, *viz.* one side and the angles given, or two sides and one angle given to find the rest; yet as they are intended to be applied to navigation, we shall divide them into six different cases as usual, and illustrate the whole by an example in each case, whereby it will be very plain, that in finding sines, tangents, or secants, to any given arch we make a right angled triangle, as in the following examples.

CASE I. Given the radius A C of 960 feet, required the sine and co-sine of an arch of 40 degrees, or which is the same thing. In the right angled triangle A B C given the hypotenuse A C 960 feet, and the angle at A 40 degrees, required A B the perpendicular, and B C the base?

1. Draw a line parallel to the margin, and make an angle of 40 degrees at the point A, by a line of chords; (as in Prob. 8.) on which lay off 960 by a scale of equal parts from A to C. *Plate II. Fig. 7.*

2. Let fall a perpendicular from C, to cut the line parallel to the margin <sup>in</sup> B; so shall A B (the sine complement of the angle at A,) be the perpendicular, and B C, (the sine of the angle at A,) the base.

CASE II. Given the radius A B, 735 feet; required the tangent and secant of an arch of 40 degrees? Or which is the same thing, in the triangle A B C, given the perpendicular A B 735 feet, and the angle at A 40 degrees, required the hypotenuse and base? *Pl. II. Fig. 7.*

1. Make the given angle at A as before, and lay off 735, from A to B.

2. Erect a perpendicular at B, to intersect the line A C, in C; so shall B C, (the tangent of the angle at A,) be the base; and A C (the secant,) the hypothenuse.

CASE III. Given the radius B C, 617 feet; required the tangent complement, and secant complement of an arch of 40 degrees? or, which is the same thing, given the base B C, 617 feet, and the angle at A, opposite thereto, required the hypothenuse and perpendicular? *Pl. II. Fig. 7.*

1. Erect a perpendicular at B, on which lay off 617, from B to C.

2. Make an angle at C, of 50 degrees, (that is the complement of the given angle) then will A C (the secant complement of the angle at A) be the hypothenuse, and A B (the tangent complement,) the perpendicular.

CASE IV. Given the radius, A C, 960, and the co-sine of an arch, A B, 735 feet; required the quantity of the arch, and the sine thereof? Or, which is the same thing, given the hypothenuse A C, 960, and the perpendicular A B, 735 feet, required the angles and base? *Pl. II. Fig. 7.*

1. Erect a perpendicular at B, and lay off 735 from B to A.

2. Take 960, the given radius, with a pair of compasses, and one foot in A, with the other cut the base in C, so the angles may be measured by a line of chords, and the base by a line of equal parts.

CASE V. Given the radius A C, 960 feet, and B C, the sine of an arch, 617; required the quantity of the arch, and co-sine thereof? Or, which is the same thing, given the hypothenuse A C, 960, and the base, B C, 617; required the angles, and perpendicular? This is the same with the preceding, only, laying off 617 from B to C, and then taking

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960 feet in the compasses, and one foot in C, with the other cut the perpendicular in A ; so the angles and perpendicular may be measured. *Plate II. Fig. 7.*

**CASE VI.** Given the radius A B, 735 feet, and tangent B C 617 ; required the quantity of the arch ? Or, which is the same thing, given the perpendicular A B, 735 feet, and the base B C, 617, required the angles, and hypothenuse ? *Pl. II. Fig. 7.*

Make a right angle at B, lay off 735 feet from B to A, and 617 from B to C, and draw the line A C, which constructs the triangle ; so the unknown parts may be measured.

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**C H A P. III.**

**S E C T. I. Trigonometry Arithmetically.**

**T**O make a table of natural sines, tangents and secants, and thereby to solve all the varieties of right angled triangles.

We have, in the preceding chapter, shewn how the solutions to all the cases of right angled triangles may be obtained geometrically, but as this would be too tedious for practice, we shall here shew how they may be very expeditiously done by the rule of three.

It is easy to observe, that, in the construction of all the preceding triangles, though the radius of a circle be given, that is not sufficient, to obtain the unknown parts, for we must always have the radius of another circle given, before we can construct the triangle, *viz.* the chord of 60 degrees, and we also have, in the scheme for constructing the sines, tangents, and secants, a triangle similar to that in question ; so that in effect we only make a triangle similar to a given one, and when the triangle is constructed,

structed, the unknown parts may be measured; now all this may be done without constructing the triangle.

In order to perform this, we must make a table where the sines, tangents, and secants of every degree and minute of the quadrant may be had by inspection. This has been performed with great accuracy, by several eminent mathematicians to a radius of 1000000 equal parts; but as in this treatise we do not explain the manner of extracting the square root, we shall omit the calculations, and satisfy ourselves with measuring the several sines, tangents, and secants that have been geometrically constructed, and collecting them as in the following table.

*A Table of natural SINES, TANGENTS,  
and SECANTS.*

Deg.	Sines.	Tang.	Seca.		
5	871	9962	875	11430	10038
10	1736	9848	1763	56713	10154
15	2588	9659	2679	37320	10353
20	3420	9397	3639	27474	10642
25	4226	9063	4663	21445	11034
30	5000	8660	5773	17320	11547
35	5735	8191	7002	14281	12208
40	6428	7660	8390	11917	13054
45	7071	7071	10000	10000	14142
	Sines.	Tang.	Seca.		Deg.

This table gives, by inspection, the sines, tangents, and secants, of every fifth degree of the quadrant of a circle whose radius is 10000. And because it was before proved, that as the radius of any circle, is to the sines, tangents, and secants of the degrees of the same circle, so is the radius of any

any other circle, to the sines, tangents, and secants, of the degrees of this last circle. Hence it is evident that if the radius of any circle be known, the sine, tangent, and secant of any arch in that circle may be found by the rule of three; or if the radius, and the sine, tangent, or secant, be known, we may, by the same rule, find the quantity of the arch. The various solutions may be obtained by the two following general proportions.

Case I. The radius and arch given to find the sine, tangent, or secant thereof, or, which is the same thing, one side and the angles, given to find the other two sides.

As the tabular radius 10000

Is to the tabular sine, tangent, or secant of any arch

So is the radius of any circle, or the given side of a triangle

To the required sine, tangent, or secant; or to the required side.

Case II. The radius, and sine, or tangent, or secant of an unknown arch being given, to find the quantity of the arch? This is the same, as if two sides of a right angled triangle were given, to find the angles. The proportion is

As the given radius, or one of the given sides,

Is, to the given sine, or tangent, or secant, or to the other given side,

So is the tabular radius

To a sine, tangent, or secant in the table.

This last number, which is the result of the operation, must be looked for in its proper column, in the table, and the degrees corresponding thereto, will be the quantity of the arch, or angle required. For, it must be observed, when an angle is required, we do not work for the angle itself, but for the sine, tangent, or secant of it.

The

The following Examples will sufficiently illustrate what has been said on this head, which, it is presumed, may be performed, either, by multiplication and division, or by the logarithms, without any further directions, than those already delivered in the rule of three, and in the operations the very same figures will be found, as in some of the examples of that rule.

Case I. Given the radius or hypothenuse 960, and arch 40 degrees, required the sine and co-sine thereof? Or, which is the same thing, the base and perpendicular?

	Logarithms.
As 10000 the tabular radius	4.00000
Is to 6428, the co-sine of 40 degrees,	3.80807
So is A C, 960, the given radius or hypothenuse	2.98227
To B C 617.1 the required sine or base	2.79034

Operation.  $6428 \times 960 = 6170680$ , the logarithm of which is (the sum of the logarithm of the second and third terms) 6.79034.

As the index is 6, the natural number corresponding thereto, will consist of seven figures, and therefore cannot be found in the table, but there will be no occasion for finding this natural number, because whatever it is, it must be divided by the first term, the quotient will then be 617.0680; and when the logarithm of the first term is subtracted from the sum of the logarithms of the second and third terms, we shall have 2.79034, the logarithm of 617.0680, nearly 617.1 and this may be had in the table, because the index is 2; so there must be but three figures of integers in the natural number, and the fourth figure will be a decimal.

Case II. Given the radius, or perpendicular A B,

735 feet, required the tangent and secant of 40 degrees, or the hypothenuse and base?

Case III. Given B C, 617 feet, the radius, or base of the triangle, required the tangent complement, and secant complement, of an angle of 40 degrees, or the perpendicular and hypothenuse?

Case IV. Given A C, 960 feet, the radius or hypothenuse, and A B, 735 feet, the sine complement, or perpendicular; required the angle and sine thereof or base?

Case V. Given A C, 960 feet, the radius or hypothenuse, and B C, 617, the sine of an arch, or the base; required the quantity of the arch, or angle, and the sine complement thereof?

Case VI. Given B A, 735, the radius, or perpendicular, and B C 617, the tangent of an arch or the base; required the quantity of the arch, or angle, and secant thereof?

As these examples have been already done geometrically in the preceding chapter, it will be found when they are done arithmetically, by the same method the first example is done by, we shall find the result agree with the former operations.

## S E C T. II. *Of placing the three given Terms in proper Order; and where they are to be found.*

In the preceding sections we have supposed the given side to be the radius of a circle; but the solutions may be obtained by making the given side a sine, tangent, or secant.

The proper names of the sides of the triangle are hypothenuse, perpendicular, and base; but as they may be considered as sines, tangents, or secants of the arches that measure the oblique angles, they will thereby acquire another name, which we shall call their sur-name. Now, as any side of the triangle may be made the radius of the arch that measures

measures the oblique angles, this will occasion their sur-names to vary according to the side made radius, for, if the hypothenuse be made radius, as in Case I, the base will be the sine, and the perpendicular the sine complement of the angle. If the perpendicular be radius, as in Case II, the hypothenuse will be the secant, and the base the tangent of the angle. If the base be made radius, as in Case III, the hypothenuse will be the secant complement, and the base the tangent complement of the angle. *Note.* By the angle is understood that opposite to the base, denoted by A, and that at C, opposite to the perpendicular, is called the complement angle. Before we come to shew how to place the given terms, it will be proper to make the two following remarks, which will enable us to discover where the given things are to be found.

*Rem. 1.* When the operation is for a side, there is only one of the given things in the triangle, which may be, indifferently, either the second or third term; but we shall always make it the third term, and so the two first terms will be in the table of artificial sines, &c. and the two last in the logarithms.

*Rem. 2.* When the operation is for an angle, there are two of the given things in the triangle, *viz.* the two given sides; their logarithms, must be the first two terms. The third term is the radius in the table, whose logarithm is, in the table of artificials, 10.000000, and when the logarithm of the fourth term is found by an operation, we must look for it in the table of artificials, in its proper column, the degrees corresponding to which is the quantity of the required angle.

Hence it is obvious that the first thing to be done before we can state the terms of the proportion, is, to chuse one of the sides for the radius,

this

this will determine the sur-names of the other two sides; so that when a side is required every case will admit of three, and when an angle is required, every case will admit of two different operations.

When a side is required, the first term must always be the sur-name of the given side; the second term the sur-name of the required side; the third term the proper name of the given side; so shall the fourth term be the proper name of the required side; from whence we have the following general proportion.

As the sur-name of the given side  
Is to the sur-name of the required side } these to be found  
in the artificials.

So is the proper name of the given side  
To the proper name of the required side } these to be found  
in the logarithms.

When an angle is required, there are always two sides given, and either of them may be made the radius of the arch that measures the oblique angles: The first term will be the proper name of that given side which is made radius. the second term the proper name of the other given side; the third term will be the sur-name of the first term, which will always be radius, and the fourth term, the sur-name of the second side; the proportion is

As the proper name of that given side, made radius  
Is to the proper name of the other given side,  
So is the sur-name of the first term, *viz.* radius  
To the sur-name of the second term.

Here the logarithms of the first two terms must be sought out of the table of logarithms, and when the logarithm of the third, *viz.* 10.000000, taken out of the artificials, is added to that of the second, and the logarithm of the first, subtracted from

their sum, we shall have the logarithm of a sine, tangent, or secant; this must be sought out of the table of artificials, in its proper column, the degrees answering to which will be the angle required.

The different proportions for the solution of all the varieties of right angled triangles are expressed in the following table.

The proportion for the various solutions of the six cases of right angled triangles.

CASE 1. Given hypothenuse and angle: required perpendicular and base?

$$\text{Radius} \left\{ \begin{array}{l} H \left\{ \begin{array}{l} R : \text{co-sine} :: H : P \text{ and } R : \text{fine} :: H : B \\ P \left\{ \begin{array}{l} \text{Sect} : R :: H : P \text{ and } \text{Sect} : \text{Tang} :: H : B \\ B \left\{ \begin{array}{l} \text{Co-sect} : R :: H : B \text{ and } \text{Co-sect} : \text{Co-tang} :: H : P \end{array} \right. \end{array} \right. \end{array} \right. \end{array} \right. \end{array}$$

CASE 2. Given perpendicular and angle: required hypothenuse and base?

$$\text{Radius} \left\{ \begin{array}{l} H \left\{ \begin{array}{l} \text{Co-sine} : R :: P : H \text{ and } \text{co-sine} : \text{fine} :: P : B \\ P \left\{ \begin{array}{l} R : \text{Sect} :: P : H \text{ and } R : T :: P : B \\ B \left\{ \begin{array}{l} \text{Co-tang} : \text{co-sect} :: P : H \text{ and } \text{co-tang} : R :: P : B \end{array} \right. \end{array} \right. \end{array} \right. \end{array} \right. \end{array}$$

CASE 3. Given base and angle: required hypothenuse and perpendicular?

$$\text{Radius} \left\{ \begin{array}{l} H \left\{ \begin{array}{l} \text{Sine} : R :: B : H \text{ and } \text{fine} : \text{cosine} :: B : P \\ P \left\{ \begin{array}{l} T : \text{Sect} :: B : H, \text{ and } T : R :: B : P \\ B \left\{ \begin{array}{l} R : \text{co-sect} :: B : H \text{ and } R : \text{co-tang} :: B : P. \end{array} \right. \end{array} \right. \end{array} \right. \end{array} \right. \end{array}$$

CASE 4. Given hypothenuse and perpendicular: required angles and base?

$$H : B$$

$$\text{Radius} \left\{ \begin{array}{l} H \left\{ \begin{array}{l} H : P :: R : \text{co-sine}, \text{ and } R : \text{fine} :: H : B \\ P : H :: R : \text{Sect. and } R : T :: P : B \end{array} \right. \\ P \end{array} \right. \right.$$

**CASE 5.** Given hypotenuse and base: required angles and perpendicular?

$$\text{Radius} \left\{ \begin{array}{l} H \left\{ \begin{array}{l} H : B :: R : \text{Sine and } R : \text{co-sine} :: \\ H : P \end{array} \right. \\ B \left\{ \begin{array}{l} B : H :: R : \text{co-sect. and } R : \text{co-tang} :: \\ B : P \end{array} \right. \end{array} \right. \right.$$

**CASE 6.** Given base and perpendicular: required angles and hypotenuse?

$$\text{Radius} \left\{ \begin{array}{l} P \left\{ \begin{array}{l} P : B :: R : T \text{ and } R : \text{seft} :: P : H \\ B : P :: R : \text{co-tang. and } R : \text{co-seft} :: \end{array} \right. \\ B \left\{ \begin{array}{l} B : H \end{array} \right. \end{array} \right. \right.$$

This table contains all the varieties that can happen in the solutions of all right angled triangles. It will be convenient, to have the radius in the given things, on account of the cyphers; and it would not be amiss to work some examples all the different ways, and it is presumed, the satisfaction of seeing the same figures to be the result of the different operations, would compensate the labour. In practice we shall have no occasion for the natural sines, and therefore they are not in the common navigation books, for if we had them we should work by their logarithms, and these we have in the table of artificial sines, tangents, and secants, they being only the logarithms of the natural. The sine of 5 degrees in the artificials is 8.940296, the nearest natural number corresponding to which is 8716; but as the index is 8, it should consist of nine figures; which shews the radius by which the table was made consisted of eleven figures, and whereas the common tables of logarithms go no further

further than four figures in the natural numbers, we took 10000 for the radius by which we made our table of natural sines, tangents, and secants, only to every fifth degree, that being sufficient to shew that the artificials are the logarithms of the natural; for if 6 be subtracted from the index of any artificial sine, tangent, or secant, we shall have the natural sine, tangent, or secant, corresponding thereto in the table of logarithms; the index of the artificial sine of 5 degrees is 8, from which taking 6, remains 2.940296; the nearest natural number corresponding to this is 871.5, which is the natural sine of 5 degrees to a radius of 10000 as in the table, and all the other natural sines, tangents, and secants, in the table, upon examination, will be found to be only natural numbers, of which the artificial sines, tangents, and secants, are the logarithms.

We have, in Sect. 4, page 16, shewn how to construct *Gunter's* line of numbers, and the manner of working any questions in the rule of three by it, and shall here shew how all the cases in trigonometry may be solved by *Gunter's* lines. If we work by the natural sines, tangents, and secants, this is performed in the same manner as before directed, by the line of numbers only; but as the artificials are all used in the calculations, Mr *Gunter* has likewise constructed a line of artificial sines and tangents adapted to the line of numbers, which are the logarithms of the natural. Against 5 degrees on the line of sines, is 871 on the line of numbers; against 10 on the line of sines is 1736 on the line of numbers, evidently the natural sines corresponding to those degrees, as in the table, and it will be the same with all the rest. The lines being thus constructed, the operations in triangles will be

the

**Sect. II. Trigonometry by Gunter's Lines.** 53  
the same as in the rule of three, and may be solved by this general rule.

1. Place the terms as before directed, so shall the first and second be of one name, and the third and fourth likewise of one name, though different from the two first terms.

2. Extend from the first to the second term, upon the line of numbers, when they are sides of the triangle, but, if the first two terms be sines or tangents, we must extend on these lines, and this extent will reach from the third term, to the fourth.  
*Note*, If the first two are on the line of numbers, the other two will be either on the sines, or tangents, and the contrary. The two following examples will sufficiently illustrate what has been said on this head.

*Example 1.* Hypotenuse 960, angle 40 degrees, required base and perpendicular?

In order to make the first and second term of one name, instead of the radius we shall take the sine of 90 degrees, so it will be sine 90 : sine 40 :: 960 : 617 base; and sine 90 : co-sine 40 :: 960 : 735 the perpendicular. The extent upon the line of sines from 90 to 40, will reach on the line of numbers from 960 to 617, and the extent from 90 to 50, (the complement of 40) on the sines, will reach from 960, on the line of numbers, to 735.

*Example 2.* Base 617, perpendicular 735 feet; required the angle?

735 : 617 :: R : T. Here the first and second terms are of one name, and to make the third and fourth of one name, instead of the radius, take the tangent of 45, and it will be 735 : 617 :: tangent 45 : tangent 40; or 617 : 735 :: tangent 45 : tangent 50. In both these proportions, the extent from

from the first to the second, on the line of numbers, will be the same, and of consequence, when laid off on the line of tangents, it will reach, in both cases, from 45 to the same point in that line.

The line of tangents on the scale is numbered from 1 to 45, which is equal to the radius, and the tangents above 45 are on this line in the same points with their complements; that is, the tangent of 46 is in the same point with that of 44; the tangent of 40 and 50 are in the same point; by this means every point in the line of tangents will stand for the tangent and co-tangent of any arch or angle; and, in order to know which of the two is that which is required, observe whether the first or second term be greatest; for, if the second be the greatest, the fourth term will be greater than the third, and therefore more than 45; but, if the second term be less than the first, the fourth term will be less than 45 degrees, as in the preceding example. When 617 is the second term, the fourth will be 40, the tangent of the angle; but when 735 is the second term, the fourth will be 50, the tangent complement of the angle.

If there be a secant in the proportion, a solution cannot be had by these lines, because no secant and the radius of the same circle can be of equal length; for the radius lies betwixt the center and circumference, whereas the secant must be produced beyond the circumference.

Having now explained all the various ways of solving right angled triangles, *viz.* geometrically, arithmetically, and by Gunter's lines, we have so far executed the first part of the plan. Nevertheless it will not be amiss to recapitulate the substance of what has been said on the whole.

1. In the rule of three there are four numbers concerned, only two of which are in the triangle; for

for it is a great mistake to imagine, because in every case there are always three parts, independant of one another, of the triangle known, that these are the terms of the proportion, no, it is evident, by all the preceding operations, that we have in constructing the sines, &c. formed a triangle similar to that in question, of which we have all the parts, so that in effect it is only making a triangle similar to a given one. (See *Theo. 6, Chap. 2*; and *Seit. II Chap. III.*)

2. As the whole of trigonometry consists in measuring the sides and angles of triangles; before this can be performed we must actually divide the circumference into degrees, and even into minutes, and from thence construct a line of chords to make or measure angles; we must likewise make a scale of equal parts to measure the sides, (see *Prob. 8 and 9*) by these two lines the geometrical solutions are performed, and in order to do the same arithmetically we must prove

3. That all the angles of any triangle make 180 degrees, that the radius of any circle is equal to the chord of 60 degrees of the same circle, (See *Theo. V. Inf. 1 and 4*, and that as the radius of any circle is to the chords of the degrees of that circle, so is the radius of any other circle to the chords of the degrees of this last circle, (See *Theo. VI. Inf. 1*). We must likewise explain the sines, tangents, and secants, and that in different circles they are in proportion to their radiusses, and that the sides of a right angled triangle may be considered as sines, tangents, and secants, and a table of natural sines, tangents, and secants calculated, before the solutions can be obtained arithmetically. (See *Seit. IV. Chap. II.*)

4. To prevent the tedious operations by multiplication and division, the nature and use of the loga-

logarithms must be explained, (See Sect. 3, Chap. I, also the construction and use of Gunter's lines. (See Sect. 4, page 16.)

We shall add one example more in each case, and to shew that navigation is performed by right angled triangles, we shall make use of the same examples when we come to plain sailing, only giving the sides different names.

Case 1. Hypotenuse 180 miles, angle  $70^{\circ} 19'$ , required perpendicular and base?

Case 2. Perpendicular 200 miles, angle  $33^{\circ} 45'$ , required perpendicular and base?

Case 3. Base 60 miles, angle  $11^{\circ} 15'$ , required hypotenuse and perpendicular.

Case 4. Hypotenuse 180, perpendicular 61 miles, required angles and base?

Case 5. Hypotenuse 240, base 133, required angles and perpendicular?

Case 6. Perpendicular 302, base 60, required angles and hypotenuse?

To render trigonometry subservient to navigation, the positions of places, and their distances from one another must be determined, which is the business of geography, the principles of which are next to be considered.

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## C H A P. IV.

### Of G E O G R A P H Y.

IT has been found by experience that our earth is not a flat extended plane; this the mariner can make no doubt of, for when he is at sea, out of sight of land, he finds himself in the center of a circle

circle, in the circumference of which the sea and sky seem to unite the heavens, at the same time, forming a concave sphere over his head. Now, suppose he sees any object, such as a ship, or small island, just appearing out of the water, and sail directly to it, he will find himself when he arrives at that place, the same distance from the circle that terminates his sight, as before ; and likewise surrounded by a concave sphere over his head, all the parts of which will be at the same distance as before, which could not possibly be, if the earth and sea together did not form a body of a globular form. This is what they call the terrestrial or teraqueous globe, and in order to give us an idea of the position of places, they chuse to lay them down upon an artificial globe, made to represent the real one in which we live.

They likewise have made another to represent the heavens, which they call the coelestial globe, for though they form a concave sphere, yet they have delineated all the constellations, and principal stars, upon the convex superficies of this globe, in such a manner, that, if it were transparent, and the earth in the center, to an inhabitant on the earth, they would all appear as they actually do as <sup>the</sup> the heavens.

### S E C T. I. *Geographical Definitions.*

*Def. 1.* A globe, or sphere, is a solid body, which has a point within it equally distant from all parts of the circumference ; it may be formed by the revolution of a semi-circle round its diameter, supposing the diameter wholly immoveable.

2. The axis of the globe is the line passing through the center, round which the artificial globe is turned : The earth likewise is supposed to move round an imaginary axis, which occasions the diurnal revolution of the sun and stars.

3. The poles are two points in the surface of the globe, through which the axis is supposed to pass; and if the earth's axis were produced both ways to the heavens, it would pass through both the celestial poles, round which the heavens, with the sun, moon, and stars, seem to move in 24 hours; one of these is called the north or arctick; the other, the south or antarctick pole. There is a star in the heavens near the north pole, which, seemingly, has little or no motion, and called the north, or pole star.

4. Meridians are great circles intersecting one another in both poles. *Note*, Great circles are such as divide the globe, and of consequence each other, into two equal parts.

5. The equator, or equinoctial, by mariners called the line, is a great circle equally distant from both poles, and therefore bisects all the meridians at right angles.

6. The ecliptick is a great circle in the heavens cutting the equinoctial in an angle of  $23^{\circ} 29'$ . The sun is always in this circle, in which he appears to us to make an entire revolution in one year, this is, what is called the sun's annual motion; and whereas the heavens seem to revolve round us once in 24 hours, the sun seems by that motion to describe a circle in the heavens every day, which is called the sun's diurnal motion.

7. Parallel circles, are small or lesser circles, which divide the globe into two unequal parts, those that are drawn upon the globe are parallel to the equator, and are called parallels of latitude on the terrestrial, but parallels of declination on the celestial globe.

These are all the circles which are actually drawn upon the globes, but there are other imaginary ones, which, though they cannot be actually drawn, because

because they vary as the observer changes his situation; yet they may be represented by the wooden frame, and other appurtenances.

8. The horizon is that circle in the heavens which terminates our sight; as, supposing the observer at sea, he always finds himself in the center of a circle, the sea and sky seeming to unite at the utmost visible extent of his sight. This is represented by the wooden frame.

9. The zenith, is that point in the heavens right over the observer's head; and that point in the heavens right opposite to it, is called the nadir: If a line were drawn from the zenith to the nadir it would pass through the center of the earth perpendicular to the horizon, and likewise pass through the center of the horizon; so the zenith and nadir are exactly 90 degrees from every point in the periphery of the horizon; this is called the rational horizon, and divides the earth into two equal parts, called the upper and the lower hemispheres: The horizon seen by us is the sensible or visible, and is always parallel to the rational, the semi-diameter of the earth being the distance betwixt them.

10. Azimuths are great circles passing through the zenith and nadir, and therefore perpendicular to the horizon, they, together with the horizon, zenith, and nadir, alter their position according to the situation of the observer, and are, in respect to the horizon, what meridians are to the equator, for if the poles be in the zenith and nadir, the equator will become the horizon, and all the meridians azimuths. The meridian of the place is an azimuth circle cutting the horizon in the north and south points, and the azimuth circle, which cuts the horizon in the east and west points, is called the prime vertical.

11. Almicanters are lesser circles parallel to the horizon

horizon, they are likewise called parallels of altitude.

12. Latitude of a place is its distance from the equator measured on an arch of the meridian ; it is either south or north, as it lies to the southward or northward of the equator.

13. Longitude of a place, is the distance betwixt the first meridian, and the meridian of the place measured on the equator. In order to determine the latitude and longitude of places, one of the meridians is graduated both ways from the equator to each pole, this is called the first meridian ; it has no longitude, because the longitude is counted from it ; there are parallels of latitude drawn thro' every tenth degree of it, on both sides of the equator. The equator is divided into 360 degrees, and meridians drawn through every tenth degree of it ; and though there be no more actually drawn upon the globe, we may suppose a meridian and a parallel drawn through any point on its surface, and these are supplied by the brazen meridian in which the globe turns ; now the latitudes are always measured on the meridian, and will be the nearest distance of any place to the equator, but the longitude of any place is measured on the equator, and will never be the nearest distance to the first meridian, except when the place is in the equator.

14. Difference of latitude is the distance betwixt the parallels of two places measured on the meridian.

15. Difference of longitude is the distance betwixt the meridians of two places measured on the equator.

16. Departure of any place from any meridian is its distance from that meridian, measured on the parallel of latitude of the place, which will be always less than the difference of longitude if the place

place has any latitude, because this is measured on the equator, whereas the departure is measured on the parallel: The departure will indeed contain the same number of degrees that the difference of longitude does, but the degrees of the departure will contain fewer miles than those of the difference of longitude. If there be two places A and B, in different latitudes and longitudes, the departure betwixt them cannot be exactly determined; for suppose A to be in 30 degrees, and B in 40, both of the same name. A E the distance betwixt A and the meridian of B, measured in the parallel of A, will be greater than F B, the distance betwixt B and the meridian of A, measured in the parallel of B; therefore it is usual to reckon *ab* for the departure in the parallel of 35 degrees, the middle latitude betwixt the two places: Of this more when applied to navigation. *Plate IV. Fig. 1.*

17. Declination of the sun or a star, is its distance from the equinoctial measured on a meridian.

18. Tropicks are two smaller circles drawn parallel to the equator at  $23^{\circ} 29'$  distant from it, they limit the sun's course; the northermost is called the tropick of *Cancer*, and the southermost that of *Capricorn*.

19. Polar circles are  $23^{\circ} 29'$  distant from each pole, that at the north is called the arctick, and that at the south the antarctick.

All the circles on the terrestrial globe are likewise described on the coelestial, for, though the heavens are a concave sphere, all the circles may be duly represented on a convex one, and, if the observer be supposed in the center of the terrestrial, and both it and the coelestial transparent, he would see all the circles of the terrestrial globe coincide with those drawn on the convex surface of the coelestial, as was before observed. The meridians would

would intersect one another in the poles in the heavens, and the celestial equinoctial would bisect all those meridians at right angles in the heavens, in the same manner as the equator in the terrestrial intersects the meridians upon it.

*Inference I.* The distance of the zenith from the equinoctial in the heavens, measured on the meridian, is the latitude of the place; for if one line be drawn from the center of the earth to the place itself, and another to that place on the earth's surface, where the meridian of the place intersects the equator, the angle formed at the center will be measured by the arch of the meridian intercepted betwixt the place and the equator, and if both these lines be produced to the heavens, the one will terminate in the zenith, and the other in the equinoctial. Now the arch in the heavens contained betwixt the zenith and equinoctial will contain the same number of degrees with that intercepted betwixt the place and the equator, which is actually the latitude of the place.

2. If the distance of any celestial object from the zenith can be obtained by any instrument, and also the declination of that object, or its distance from the equinoctial, we may then find the latitude of the place, as shall be illustrated by various examples in another place.

3. The greatest latitude cannot exceed 90 degrees, and the greatest difference of latitude cannot exceed 180.

4. If two places be on the same side of the equator, but, on different parallels of latitude, their difference of latitude will be found by subtracting the one from the other; but if they be on different sides of the equinoctial, their difference of latitude is the sum of both latitudes added together.

5. In sailing towards the equator we decrease, but

but in sailing from the equator we increase the latitude, so that if the latitude sailed from be known, suppose A, in 50 degrees north, and the difference of latitude be likewise known, suppose 600 miles, that is 10 degrees southerly, here we are sailing towards the equator, therefore subtracting ten degrees, the difference of latitude from 50 degrees, we have 40 degrees the latitude of B; but if the difference of latitude were northerly, it must be added to 50 degrees, which gives 60 degrees the latitude come to. When the difference of latitude exceeds that sailed from, it is plain we cross the equator, and come to a contrary latitude, which is found by subtracting the latitude sailed from, out of the difference of latitude; for let that sailed from be 20 degrees north, and the difference of latitude 30 degrees southerly, the latitude come to would be 10 degrees south.

6. The height of the pole above the horizon is equal to the latitude of the place, for it is evidently equal to the distance of the equator from the zenith.

Though the earth is a sphere, and all the places on its surface most naturally described on a globe, it is more convenient, especially for navigation, to describe them on planes; this is what is called the projection of the sphere, which is the next thing to be done.

## S E C T. II.

### *The projection of the Sphere on a Plane.*

There are several ways of projecting solids upon a plane, we shall only treat of the orthographic, because all that is necessary in navigation is in this projection, performed by strait lined right angled triangles.

The manner of projecting any solid orthographically

phically on a plane, is by supposing the body to be cut into two or several parts by one or more planes parallel to one another; now, if a plane cut the body into two parts, the plane may be so far extended that each part of the surface may be represented on this plane, by letting fall perpendiculars from each point of the surface to the plane; these lines are called the projecting lines, and will be all parallel to one another; the point where these lines meet the plane of the projection, will give the position of these places on that plane. *Note*, the thing to be projected is called the original, and the plane on which it is to be projected, the plane of the projection.

Let us then suppose the globe to be cut by a plane passing through both poles, it will likewise pass through the center, and of consequence divide it into two equal parts, each being one half of the globe: Being thus cut it will lay flat on a table; the bottom on which it stands will be a circle, in this case a meridian, whose diameter will be equal to that of the globe. After being thus cut and laid flat upon a plane, if perpendiculars be let fall from all places on its surface to the plane on which it stands, we shall have one half of the globe projected, all within the circle, and, as it is by the latitudes and longitudes the situation of places are determined, we may project all the meridians and parallels on this plane, and the places may be laid down according to their true latitudes and longitudes; now all this may be performed with greater exactness, without cutting the globe, by the following method; it was necessary to observe this, to understand the principles from which the operations are deduced.

1. Describe the circle  $P\ E\ S\ Q$ , to represent the meridian; let  $P$  be the north,  $S$  the south pole;  $P\ S$  the axis,  $E\ Q$  the equator, for though it is only  
the

the diameter of it, yet, as the plane on which half of the globe now stands is a meridian, the half of the equator would be on the surface, and if perpendiculars were let fall from each point of it, they would all fall on the line E Q, and for the same reason all the parallels of latitude will be represented by their diameters, and drawn parallel to the line E Q. *Plate 4 fig. 1. 2.*

2. Graduate the equator into degrees, by laying a ruler over the several divisions of the circumference, parallel to the line P S; the points where the ruler intersects the line E Q will graduate it into degrees, and, though they are equal on the globe, they will here be unequal, for they are the sines of the arches.

The meridians are all circles on the globe equal to the equator, but as they intersect one another in the poles, they cannot be represented by strait lines on the plane of the meridian; they will all become curves when projected on this plane, but not circles. One of them which is at right angles to the plane of the projection is a strait line, and here represented by the line P S, the earth's axis: In order then to find the points in each parallel through which the meridians must pass, the parallels must all be divided into degrees; now the parallels here are represented by their diameters, as well as the equator by its diameter, they must therefore be divided into the same number of parts, and into the same proportion with the diameter of the equator, already properly divided; and to divide the diameter of each parallel into the same proportion, let us (by *Prob. 9, Chap. II.*) make the radius of the equator the side of an equilateral triangle A B C, so shall each side of the triangle be equal to the radius of the equator. Then lay off the several parts of the radius of the equator, upon the lines

K

A B

A B, and A C; so shall C, 10, C, 20, in the plane of the projection, be the same with A, 80, A, 70, in the line A B, and in the line A C, the sides of the triangle. Now when lines are drawn from the points 10, 20, &c. in the line A B, to the same points in the line A C, they will be equal to the radiuses of the several parallels, and all parallel to B C, the base of the triangle; all that remains to be done is to transfer the divisions of the radius of the equator to B C, the base of the triangle, and then lines drawn from A to the several divisions of the base, will divide all the parallels into the same proportion with the equator.

Having found all the points, the meridians may, by a steady hand, be drawn through these points, for as they are cliſes they cannot be drawn with the common compasses, which is an objection to this projection on the plane of the meridian; but this obstacle is entirely removed, because, in the cases we shall make use of it, we shall have no occasion to draw the curves, only to find the place in any parallel of latitude where any assigned meridian shall intersect it; or, if that be known, to find where the meridian will intersect the equator. However we shall shew how the meridians may be projected into strait lines by the same principles.

Let the globe be cut by a plane passing through the equator, then laid flat upon a plane, the section will be a circle, and the base or bottom on which it stands will be the equator, and, when the half of the globe is in this position, if a perpendicular be let fall from the pole, it will come right in the center of the circle, and, as all the meridians intersect one another in the pole, they will, in this projection, be strait lines, and the parallels of latitude will be circles: The radiuses of each will be the same in both projections, that is, the sines of their

their complements. Let the line A P C be the diameter of the first meridian, and P A the radius properly divided into degrees, which will thereby become a line of sines; and when circles from P, as center, are drawn through the several divisions 70, 80, &c. they will all be parallel to the equator, and therefore parallels of latitude, and it is evident their radiusses will be the sines of 10, 20, &c. the complements of their latitudes in this projection. In the former projection G H is the radius of the parallel of 40 degrees, evidently the sine of 50 degrees, the complement of the latitude.

The meridians and parallels being thus drawn, places may be laid down according to their latitudes and longitudes taken out of the tables. *Plate IV. Fig. 3.*

Let C E be 60 miles, equal a degree of the equator; draw the lines P E and P C, so shall the distance betwixt these lines in each parallel be one degree, and when measured by the same scale of equal parts, that the 60 miles in the equator was taken from, we shall have the number of miles that make a degree in any of these parallels of latitude: This is the very same thing, as if the difference of longitude betwixt two places in one parallel of latitude were given, to find their departure, that is, their distance in that parallel: And to find it by calculation, the proportion will be, As the radius of the equator is to the radius of the parallel; (or, which is the same thing, the sine-complement of the latitude,) so is their difference of longitude to their departure; or, if the departure be given to find the difference of longitude, it will be, As the radius of the parallel (or co-sine of the latitude) is to the radius of the equator, so is the departure (or distance betwixt two meridians in any parallel of latitude, to the difference of longitude,

the equator does, for which there is no remedy in the plain chart, but as this chart is absolutely necessary in navigation, we shall here shew how it may be constructed, and afterwards, how the errors in the plain are corrected by *Mercator's chart*.

## S E C T. I.

*Construction of the plain Chart.*

This may be made to contain the whole or any part of the earth, but as the parallels of latitude are equal to the equator, it will be sufficient for our purpose to make one from the parallel of 50, to that of 61. To perform this, there is no occasion to suppose the earth to be a flat extended plane, but only to represent the equator by a strait line; and because the meridians are all perpendicular to the equator, if they be likewise represented by strait lines, this will, of necessity, occasion them to be parallel to one another, the thing that is proposed. *Plate V. Fig. 2.*

Draw the line *YZ* to represent the parallel of 50 degrees, and erect the perpendiculars *WY*, and *XZ*, to represent two meridians, and in this it will agree with the globe, because the meridians intersect all the parallels at right angles.

2. By any convenient scale of equal parts, lay off 660 from *Y* to *W*, and from *Z* to *X*, and draw the line *WX* which will be the parallel of 61 degrees, for 11 degrees is 660 miles.

3. Divide the meridians into degrees; thus, take 60 by any line of equal parts, which lay off on the lines *YW*, and *ZX*, to 51, and from 51 to 52, &c. and where the degrees will admit, they may be subdivided into halves, and quarters, or into miles.

4. Gra-

4. Graduate the parallels W X, and Y Z, into degrees, making them equal to those of the meridian, and draw the line A V to represent the first meridian, from whence the longitude is to be accounted.

The chart being thus limited, we may lay down places upon it by their latitudes and longitudes; but it must be observed that the departure and difference of longitude on this chart are the same thing.

Let it then be required to lay down the following places according to their latitudes and longitudes, as follows.

Latitude North.    Longitude West.

	Deg.	Min.	Deg.	Min.
B	51	1	2	49
C	54	21	5	02
D	59	23	6	02
E	61	00	6	02
F	59	59	3	13
G	56	39	1	00
H	51	37	0	00
A	50	00	0	00

In order to lay down these, first draw a line parallel to Y Z, 61 miles distant from it. Now it is certain B must be somewhere in that line, and because it is in West longitude, lay off 169 miles from A and V, towards Y and W, or, which is the same thing, 49 miles from the second degree of longitude, and then draw a meridian, which will intersect the parallel in B: In like manner lay off 21 miles from 54 deg. on both the meridians, and there draw another parallel of latitude; then lay off 2 miles on the parallels W X, and Y Z, from the 5th degree

degree of longitude, and draw a meridian to intersect the parallel in C, all the rest may be laid down by this general rule. First, look for the latitude on both the graduated meridians, where draw a parallel of latitude. Secondly, look for the longitude on the graduated parallels, where draw a meridian, which will intersect the parallel of latitude in the place required: Or, if the places be already laid down, their latitudes and longitudes may be found by drawing a parallel and meridian through each, to intersect the graduated meridians and parallels, which will give their latitudes and longitudes: Or, by taking, with a pair of compasses, the nearest distance of the place from any parallel of latitude, and laying it off from that parallel on the graduated meridian, and this will give the latitude: The longitude is found by taking the distance of the place from any meridian, and laying it off from the same meridian on the graduated parallel.

The places being thus laid down, if we draw a line from A to B, from B to C, &c. we shall have so many right angled triangles. Hence it will be easy to apply the doctrine of right angled triangles to navigation, for, it is evident, that upon the plain chart the distance, difference of latitude, and departure, always makes a right angled triangle, the distance answers to the hypotenuse, the difference of latitude to the perpendicular, and the departure to the base, the course is the angle which the rhumb line the ship sails upon makes with the meridian, and is obtained by the mariner's compass, as was before observed. The distance is had by the log line, which must be very carefully divided, so that every knot may contain exactly the 120th part of an hour, for then the ship will go just as many miles in an hour as there are knots of the line run out in half a minute. It will not be easy to determine

mine how many miles will make one degree ; this, on land, has been done by actual mensuration, by several eminent mathematicians, to whom recourse must be had for determining this point ; but as the mariner may have frequent opportunities of sailing on the meridian, if then the observed latitude agrees with that by account, it is probable the line is truly divided, but if there be any considerable difference, it may be presumed the fault must be in the line, for the glass may be adjusted exactly to 30 seconds ; and if, upon frequent trials, the same error is found, the line may then be truly divided. But, as the course cannot be depended upon with certainty on account of currents, it is no wonder the mariner often falls into great errors.

*The Resolution of the Six Cases of Plain Sailing.*

As these will be so many right angled triangles, we shall refer to the six cases in trigonometry for their construction ; and here only remark, that, as the angle is given in points of the compass, it, for the most part, will consist of degrees and odd minutes, which cannot be taken off the line of chords exactly, for which reason there is a line of rhumbs to be used, instead of the chords, for setting off the course ; this is only a line of chords, the quadrant being first divided into eight equal arches, and these subdivided into halves and quarters, and from thence transferred to the chord of 90 degrees, which will be equal to eight points. The line being thus constructed, before it can be used for the course, we must first describe an arch with the chord of 60 degrees, taken from the line of chords, to which the rhumb line is adapted. Again, as in triangles, the perpendicular is drawn parallel to the margin, it will be convenient to draw the difference of latitude so also ; and of consequence the

departure will be parallel to the top and bottom, as the base is in Trigonometry; for in all books (where it is not otherwise expressed) east is on the right, and west on the left hand, the top north, and the bottom south; so that in sailing betwixt N and W, the angle of the course must be at the bottom on the right hand; but if between the N. and the E, the angle of the course must be at bottom, on the left hand: If the course be southerly, it must be on the top when easterly; but on the right hand when westerly.

In projecting the following examples, it will be proper to draw the equator in each; so then we may compare the latitude found by projection with that by calculation. *Pl. II. Fig. 9.*

**CASE I.** A ship in 16 deg. 10 min. N. lat. sails S W by S  $\frac{1}{2}$  W 960 miles; required the latitude come to and departure?

1. Draw the line E Q to represent the equator, and because we are sailing on the S W quarter, erect a perpendicular on the right hand at Q.

2. Lay off, by a scale of equal parts, 970 miles, that is 16 degrees 10 minutes from Q to A, which must be the latitude, the ship sails from.

3. With the chord of 60 degrees from the center A, describe an arch, on which lay off  $3\frac{1}{2}$  points, that being the course.

4. Draw the line A C, on which lay off 960 miles, the given distance from A to C; and, lastly, let fall a perpendicular from C, to cut the meridian in B; so shall B C be the departure, A B the difference of latitude, and B Q equal C E, the latitude come to.

**CASE II.** A ship in 3 degrees, 48 min. N lat. sails N E by N  $\frac{1}{2}$  E, till she arrives in the latitude of 16 degrees 10 minutes N, required the distance and departure?

This

This is the same as the 2d in Trigonometry, and because the course is just as many points as in the foregoing, and the latitude sailed from, here, is that come to before, it is plain the triangle will be exactly the same as in the preceding; and if we draw  $D E$  equal and parallel to  $A Q$ , and  $D A$  parallel to  $C B$ , we shall have the triangle properly projected; so shall  $A Q$ , equal  $D E$ , be the latitude come to,  $C A$  the distance, and  $D A$  the departure. *Pl. II.*  
*Fig. 9.*

**CASE III.** A ship in 16 degrees, 10 minutes, N latitude, sails S W by S  $\frac{1}{2}$  W, till her departure is 609 miles; required the distance and latitude come to?

This may be constructed as the 3d case in Trigonometry, but we shall here shew how it may be done by another method.

1. Draw the equator  $E Q$ , meridian  $Q A$ , on which lay off the given latitude from  $Q$  to  $A$ , and make the course at  $A$ , as in the first case.

2. Draw  $A D$  parallel to  $E Q$ , on which lay off 609, the departure, from  $A$  to  $D$ .

3. Draw a meridian through  $D$ , to intersect the rhumb line, drawn from  $A$ , in  $C$ ; lastly, let fall the perpendicular  $C B$ , which will be the given departure,  $A C$  the distance,  $A B$  the difference of latitude, and  $B Q$  equal  $C E$ , the latitude come to. *Pl. II.*  
*Fig. 9.*

**CASE IV.** A ship in 4 degrees 16 minutes N latitude, sails betwixt the S and the W till the distance is 960, and difference of latitude 742 miles, required the course and departure?

Here having assumed  $A$  for the place sailed from, and constructed the triangle as in the like case of Trigonometry, we may lay off 256 miles, that is 4 degree 16 minutes, from  $A$  to  $Q$ , so shall  $E Q$  be the equator, and  $Q B$  the latitude come to south,

*Pl. II.*

*Pl. II. Fig. 7.* Note, The latitude come to, is found, by taking the latitude sailed from, out of the difference of latitude, which will always be the case, when the difference of latitude exceeds that sailed from, as here when the ship has made 256 difference of latitude southerly, it is plain she comes to the equator, and what more difference of latitude she makes will be in south latitude.

CASE V. A ship in 8 degrees 6 minutes, S latitude, sails betwixt the N and E till her distance is 960, and departure 609; required the course and latitude come to? *Pl. II. Fig. 7.*

This is projected as in Trigonometry, and the equator found as in the preceding, and because the difference of latitude exceeds that sailed from, we find the latitude come to as in the preceding.

CASE VI. A ship in 4 degrees 16 minutes N latitude, sails betwixt the S and W till her difference of latitude is 742, and departure 609; required the course and distance? *Pl. II. Fig. 7.*

It would be quite needless to give any instructions for the projecting this, as the triangle is the same with the preceding; we shall therefore work the first case arithmetically, but it is to be observed that the course must be converted into degrees before the operation can be performed: Now here the course is 3 $\frac{1}{2}$  points, or 39 degrees 22 minutes.

As radius	10.00000
Is to co-sine 39° 22'	9.88824
So is distant 960 miles	2.98227
To different latitude nearly 742	2.87051

As radius	10.00000
Is to sine of 39° 22'	9.80228
So is distant 960 miles	2.98227
To departure nearly 609	2.78455

In

In the preceding cases, one triangle serves for all; we shall therefore subjoin the following, and, though they may be projected as in Trigonometry, we think it proper here to remark, that the first thing to be done is to draw a meridian; and when there is only one triangle concerned, it is indifferent at what point of the meridian the course be made; when there are more than one triangle concerned, the point in the meridian, for the first course may be assumed, where most convenient, but then this will determine the points for the other courses, in the several meridians. As, for instance, in sailing from A to B, from B to C, &c. on different courses; the point A may be any where, so, A B, will be the distance; but, in sailing from B, the point for the course is fixed, therefore we must draw a meridian through B, before we lay off the course, and when C is found we must likewise draw a meridian there, and lay off the course to D, &c. This will be best understood by the following examples, which contain all the varieties of plain sailing, and the substance of what is commonly called traverse.

Though all the triangles are already constructed on the plain chart, we shall once more go through the whole process, and, as we intend here to shew the manner of projecting a traverse, make use of the following method.

CASE I. A ship at A, in 50 degrees N latitude, sails W N W  $\frac{1}{4}$  W 186 miles to B; required the departure and latitude of B?

1st. With the chord of 60, from the center A, describe a quarter of a circle, or, if need be, a whole or semi-circle, on which lay off the given course, and draw the rhumb line A B.

2dly, Lay off the given distance from A to B.

3dly, Draw a parallel of latitude through B to cut the meridian in 1; so shall A 1 be the

the difference of latitude, and  $B$  is the departure.  
*Plate V. Fig. 2.*

CASE II. From the same  $B$ , she sails N W by N to  $C$ , her difference of latitude is 200; required the distance, departure, and latitude of  $C$ ?

1. Draw a meridian through  $B$ ; and, in order to make the angle of the course, lay off 3 points upon the arch from the meridian of  $A$  and draw the line  $Ax$ , and another line parallel thereto, through  $B$ . It is plain this will be a N W by N rhumb line.

2dly, Lay off the given latitude, from  $B$  to 2, through which point draw a parallel of latitude to intersect the rhumb line in  $C$ , so shall  $BC$  be the distance,  $C$  2 the departure. *Pl. V. Fig. 2.*

CASE III. From  $C$  she sails N by W to  $D$ , till her departure is 60 miles; required her distance, and latitude of  $D$ ?—She then sails 97 miles N to  $E$ .

1st. Draw a meridian through  $C$ , and produce the parallel of latitude  $C$  2, to the left, till  $Cz$  is equal to the departure.

2dly, Draw a meridian through  $z$ .

3dly, Lay off one point, the given course upon the arch, from the meridian of  $A$ , and draw the line  $Ay$  and another parallel thereto, through the point  $C$ , to intersect the meridian drawn through  $z$  in  $D$ , through which point draw a parallel of latitude to intersect the meridian of  $C$  in 3, so shall  $CD$  be the distance,  $C$  3 the difference of latitude, and  $D$  3 the given departure. Then lay off 97 miles from  $D$  to  $E$ , because the course is on the meridian.  
*Pl. V. Fig. 2.*

CASE IV. A ship at  $E$  sails betwixt the S and E 180 miles, alters her latitude one degree; required the course, departure, and latitude, of  $F$ , the place come to?

1st. Lay

1st. Lay off the given difference of latitude from E to 4, through which point draw a parallel of latitude.

2dly, Take 180, the given distance with the compasses, and placing one foot in E, with the other cut the parallel of latitude in F, so shall F 4 be the departure, and E F the distance, being parallel to A B makes the course  $6 \frac{1}{2}$  points, that is S E by E. *Pl. V. Fig. 2.*

**CASE V.** From F she sails betwixt the S and E to A, her distance is 240, and departure 133 miles; required the course and latitude of A?

1st. Draw a meridian through F, and produce the parallel of latitude 4 F to the right, till F v is equal to the given departure, draw also a meridian through v.

2dly, Take the distance as before, and one foot in F, with the other cut the meridian of v in G, through which draw a parallel of latitude to cut the meridian of F in 5, so shall G 5 be the given departure, F 5 the difference of latitude, and F G the given distance parallel to A x, therefore the course will be S E by S. *Pl. V. Fig. 2.*

**CASE VI.** From G she sails betwixt the S and E to H, till she alters her latitude 5 degrees 2 minutes, and her departure 60 miles, required the course and distance and latitude of H? Then she sails 97 miles S, required the latitude come to, and the departure from the meridian of A?

1st. Draw a meridian through G, on which lay off the given difference of latitude to 6.

2dly, draw a parallel of latitude through 6, on which lay off the given departure to H, and draw the line G H, which will be the distance, and because it is parallel to A y, the course will be S by E, and then, because she sails S, lay off 97 miles on the

the meridian from H, which brings her back to A, the place first sailed from. *Pl. V. Fig. 1.*

It is plain when the triangles are constructed, the ship returns to the same place; whereas she will be above a degree to the eastward of it, as will appear in the next section. Those who incline to amuse themselves by projecting traverses may do it very expeditiously by parallel rulers, but they will unavoidably be led into very great errors, for, by this method, of solving a traverse, they cannot find the latitude and longitude the ship arrives at, and therefore cannot direct her course to the intended port, which is the very thing required. As there are only three different triangles it will be sufficient to work for the three first. In the following proportions we shall make the given side radius, and as it is all cyphers except the index, it requires no operation, only to subtract 10 from the index of the sum of the logarithms of the first and second terms.

## CASE I.

Co-sine course 70 deg. 19 min.	9.52740
Distance 180 miles	2.25527
<u>I</u> different latitude 60.6 miles	<u>1.78267</u>
Sine course 70 deg. 19 min.	9.97385
Distance 180 miles	2.25527
Departure 169.5	2.22912

## CASE II.

Sect-course 33 deg. 45 min.	10.08015
Different latitude 200 miles	2.30103
Distance 240.4 miles	2.38118
Tang. course 33 deg. 45 min.	9.82489
Different latitude 200 miles	2.30103
Departure 133.6 miles	2.12592

CASE

## CASE III.

Co-sect course 11 deg. 15 min.	10.70976
Departure 60 miles	<u>1.77815</u>
Distant 307.6	2.48791
Co-tang. course 11 deg. 15 min.	10.70134
Departure 60 miles	<u>1.77815</u>
Different latitude 301.6	2.47949

It would be altogether needless to work for the other three cases, for they are only the reverse of the precedings, as will appear when worked by the rules delivered in trigonometry; we shall therefore proceed to shew how the defects in the plain, may be supplied by *Mercator's chart*.

## S E C T. III.

*The Principles of Mercator's Chart, by Mr. Wright's Meridional Parts.*

Mr *Wright* considering that, in the plain chart, each parallel of latitude is enlarged beyond its due measure, occasioned by the meridians being all parallel to one another; and likewise that, if the meridians were made to intersect in a point, the rhumb line would make unequal angles with them; which is directly contrary to the angles, the ship actually makes on the sea, in steering by the compass. He therefore keeps the meridians still parallel to one another as in the plain, and enlarges the degrees of the meridian in the same proportion. Here then consists the difference betwixt the two charts. In the plain, the degrees of the meridian are every where equal to one another, and to those of the equator, or (which is the same thing) to those of the parallels of latitude. In *Mercator's chart*, the degrees of the equator are equal to those of the parallels of latitude, but the degrees of the meri-

dian are unequal, being enlarged, as they approach the poles, in the same proportion the parallels of latitude are enlarged: so the only difficulty will be to graduate the meridians. *Pl. II. Fig. 8.*

The first thing to be done is to find how many miles will make a degree in any parallel of latitude, the proportion is, as C B, the radius of the equator, is to D A, or C H, the radius of the parallel, or sine complement of the latitude, so is 60, the miles in one degree of the equator, to the miles in one degree of that parallel; this in the parallel of 50 deg. is, by the following operation, 38.17 miles. Now, though this makes one degree in the parallel of 50 degrees on the globe, yet, upon the chart, there are 60 miles in a degree, in that parallel, by which it is enlarged on the chart; therefore the degree of the meridian at that parallel of latitude must be likewise enlarged in the same proportion; so that by this method, there must always be two proportions, as follows,

1st, As radius of the equator 10.000000

Is to the radius of the parallel, or cosine of 50 deg. 30 min. the latitude 9.803510

So is 60, the miles of one degree in the equator, 1.778151

To 38.17, the miles in one degree of the meridian, at the parallel of 50° 1.581661

2d. As 38.17, the miles in a degree in the parallel on the globe, 1.581661

Is to 60, the miles in one degree of the meridian on the globe, 1.778151

So is 60, the miles in a degree in the parallel on the chart, 1.778151

To 94.33, the miles in one degree of the meridian on the chart 3.556302

1.974641  
Now

Now D.A, or C H the co-sine, is to C A the radius, as C B, the radius, is to C E, the secant; and D A on the chart, is always equal to D F, or C B, therefore a degree of the enlarged meridian must always be equal to the secant of the latitude, which may be found by one proportion, as in the following operation.

As radius

10.

Is to the secant of 50 deg. 30 min.

10.19649

So is 60, the miles in one degree of the  
meridian on the globe,

1.77815

To the miles in the meridian betwixt 50

and 51 on the chart

1.97464

The result is the same, as by the two precedings, and it is by this last proportion that Mr *Wright* with great labour and accuracy calculated his table of meridional parts, to every degree and minute of the quadrant from the equator to the pole.

In this table we have, by inspection, the miles (which he calls the meridional parts) that every parallel of latitude is distant from the equator, and of consequence we have the meridional parts, or the miles that are in the meridian, betwixt any two parallels of latitude by subtraction.

As we intend to construct the chart from the parallel of 50 to that of 61 degrees, we shall work for the miles in each degree of the meridian, both by the co-sines and by the secants, as in the anexed table, in which there are five columns; the first for the degrees of latitude, the second their co-sines; now, in order to find the miles in a degree of any parallel, the logarithm of 60 must be added to that of the co-sine, and the logarithm of the radius subtracted from their sum, as in the first proportion of the preceding operation, the result of which is performed in the third column, which will be

the

84 Construction of the Table of Meridional Parts.

the logarithm of the miles in a degree in that parallel: But there will be no occasion to look for the natural number corresponding to that logarithm, because, whatever it is, when we come to work for the miles in a degree of the meridian, we must subtract that logarithm from twice the logarithm of 60, as in the 2d proportion foregoing. Now, as in the third column, we have the logarithm of the miles in a degree of the parallel, so, in the fourth, we have the remainder after subtracting that logarithm from twice the logarithm of 60; and in the fifth, the natural number corresponding thereto, which is the miles in a degree of the meridian at that parallel of latitude: and this is the reason when we are to find the miles betwixt the parallel of 50 and 51, we first work for the miles in the parallel of 50 deg. 30 min. which agrees with the operation by the secants, and likewise with the table of meridional parts. The meridional parts of 50 deg. is 3474.50, the meridional parts of 51 deg. is 3568.83, so there will be 94.33 miles betwixt the parallels of 50 and 51, as by calculation; for by the table the parallel of 50 is 3474.5 miles from the equator, to which adding 94.33 we have 3568.83 miles, the distance of the parallel of 51 from the equator; and when the several secants to a radius of 60 miles are added together, their sum will be equal to the meridional parts betwixt the parallel of 50 and of 51.

*Construction of the Table of Meridional Parts.* 85

Latitude	Co-sines	Sum of cosine, and log. of 60	Remainder	Miles in the meridian.
Deg.	Min.			
50 30	9.80351	1.58166	1.97464	94.3
51 30	9.79415	1.57230	1.98400	96.4
52 30				98.6
53 30				100.9
54 30				103.3
55 30	9.75313	1.53128	2.02502	105.9
45 30				108.8
57 30				111.7
58 30				114.8
59 30				118.2
60 30	9.69234	1.47049	2.08581	121.9
Miles from the parallel of 50 to 61				1174.8

Deg.	Min.	Secants.	Log. of 60 is 1.77813
50	30	10.19649	1.97464
51	30	10.20585	1.98400

$$\begin{array}{r} \text{Latitude } 61 \text{ degrees} \quad 4649.3 \text{ merid. parts.} \\ 50 \quad \underline{3474.5} \\ \hline 1174.8 \end{array}$$

This table is inserted to shew how the table of meridional parts may be constructed ; and it is presumed, our readers may fill up the intermediates which are here omitted, and likewise continue the process by the secants, which will exactly agree with that by the co-sines, but, in practice, there will be no occasion for any of the preceding operations, as the table performs it by subtraction, or addition, as before observed.

*Construction of MERCATOR's Chart.*

1. Draw the line Y Z, to represent the parallel of 50, and graduate it as before in the plain chart, allowing 60 miles to a degree ; erect also the two meridians

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meridians Y W, and Z X, and A V to represent  
the first meridian. *Plate VI. Fig. 1.*

2dly, Lay off 94.33 miles, on each meridian,  
from 50 to 51; 96.39 miles, from 51 to 52;  
proceed, in the same manner, till you come to the  
parallel of 61, and draw the line, W X, which gra-  
duate into degrees as the parallel of 50.

The chart being thus constructed, the places may  
be laid down as in the plain chart; and, as there are  
two sea charts, navigation is generally divided in-  
to two parts, *viz.* plain and *Mercator's* sailing;  
The first we have already explained, and come now  
to *Mercator's*.

## S E C T. II.

### *Of MERCATOR's Sailing.*

In this, strictly speaking, there can be but two  
cases; first, both latitudes, which must always, be  
given, and either the course or departure, thereby  
to find the difference of longitude: Or, secondly,  
both latitudes and the difference of longitude given,  
to find the course and distance; both which are per-  
formed by right angled triangles, and may be  
projected by the two following general rules.

CASE I. Both latitudes, and either the course or  
departure given, to find the difference of longitude.

1st, Construct the triangle, as in the plain chart,  
and produce the distance and difference of latitude.

2dly, Find the meridional difference of latitude,  
which lay off on the proper difference of latitude pro-  
duced; and then, at that point, draw a line parallel  
to the departure, to intersect the distance produced;  
which will be the difference of longitude; so shall  
the meridional difference of latitude be the per-  
pendicular, and the difference of longitude, the base  
of a right angled triangle.

CASE II.

CASE II, Both latitudes, and longitudes given; to find the course and distance

1st. Make the difference of longitude the base, and the meridional difference of latitude the perpendicular of a right angled triangle; so shall the angle the hypotenuse makes with the perpendicular be the course, which suppose at the point A.

2. Lay off the proper difference of latitude from A to 1, and draw a line through 1, parallel to the difference of longitude, to intersect the hypotenuse in B; so shall A B be the distance, and B 1 the departure. *Plate VI. Fig. 1.*

The arithmetical solutions are obtained by the same proportions as in trigonometry.

It was before observed in CH. IV. SECT. 2. that the cosine of the latitude is to the radius as the departure is to the difference of longitude; and, it was proved, that the co-sine of the latitude is to the radius, as the radius is to the secant of the latitude, or, which is the same thing, as the proper difference of latitude is to the meridional difference of latitude; therefore in finding the difference of longitude, say, as the proper difference of latitude is to the departure, so is the meridional difference of latitude to the difference of longitude; or, as radius is co-tangent of the course, so is the meridional difference of latitude, to the difference of longitude.

The difference of longitude may likewise be found by supposing the departure to be made in the middle latitude; the proportion is, As the co-sine of the middle latitude is to the radius, so is the departure to the difference of longitude, and when the different latitude is not above two degrees, it will agree nearly with that by the other proportions.

*Note.* By departure is to be understood the easting, or westing, made in sailing on one direct course.

Though

Though all *Mercator's* sailing may be comprehended in the two preceding cases, we shall work for the difference of longitude to each of the six foregoing cases in plain sailing, and shall construct the triangles on *Mercator's* chart as before. *Pl. VI. Fig 1.*

**CASE 1.** After constructing the triangle, as on the plain chart, the ship comes to B: so A 1, is the proper difference of latitude; the latitude come to is 51 deg. 1 min; the meridional parts by the table is 3570.4, and the meridional parts of 50 deg. is 3474.5, which makes the meridional difference of latitude 95.9; lay this off on the meridian from A to H; through H draw a parallel of latitude to intersect the distance produced in I; so shall HI, be the difference of longitude.

**CASE 2.** 1st, Draw a meridian through I, and find the meridional difference of latitude as before, which lay off from I to a; and through a draw a parallel of latitude.

2dly, Make the angle of the course at I, or which is the same thing, through I draw a line parallel to B C, to intersect the parallel of latitude drawn through a in K; so shall a K be the difference of longitude.

**CASE 3.** 1st. Draw a meridian through K.

2dly, Lay off the meridional difference of latitude from K, to b, and there draw a parallel of latitude.

3dly, Lay off the course from K, or draw a line parallel to C D, to intersect the parallel of latitude in L; so shall b L, be the difference of longitude, now she is in 61 latitude, therefore draw a meridian through L to M.

**CASE 4.** 1st. Draw a meridian through M, on which lay off the proper, and meridional difference of latitude, and draw a parallel of latitude through each.

2dly,

2dly, Produce the distance to N; so shall N *c*, be the difference of longitude. *Pl. VI. Fig. 1.*

CASE 5. Draw a meridian through N, on which lay off the meridional difference of latitude, from N, to *d*; and there draw a parallel of latitude to intersect the distance produced in O; so shall O *d* be the difference of longitude. *Plate VI. Fig. 1.*

CASE 6. Draw a meridian through O, on which lay off the meridional difference of latitude from O to *e*, where draw a parallel of latitude to intersect the distance produced in P; so shall P *e* be the difference of longitude; and because she then sails south to the parallel of 50, draw a meridian through P to Q; so shall Q be the place the ship arrives at on *Mercator's*; and A on the plain chart. Now, to find the distance from Q to A; Q A, the difference of longitude on the parallel of 50 is given; therefore, make an angle of 40 degrees with the parallel of latitude at the point Q, because that is the complement of the latitude; so shall the perpendicular A *f* be the distance, or, which is the same thing, the departure, for in the triangle Q A *f*, if the hypotenuse Q A be the radius, then will A *f* be the co-sine of the latitude; and as radius is to cosine latitude; so is Q A, the difference of long. to A *f*, the departure; that is to say, the distance in the parallel. We have, in the plate, drawn the several departures parallel to the differences of longitudes, by which the true distances may be found on *Mercator's* chart.

CASE 7. Q in 50 degrees, N latitude and 1 deg. 45 min. E longitude; M in 61 deg. N latitude and 9° 59' W longitude; required the course and distance from Q to M? *Plate VI. Fig. 1.*

1st, Find the difference of longitude, by adding the two longitudes together, and the meridional difference of latitude by subtracting the meridional

parts of 50 deg. from the meridional parts of 61  
find also the proper difference of latitude.

2dly, Lay off the meridional difference of latitude  
from Q to  $b$ ; where draw a parallel of latitude, on  
which lay off the difference of longitude from  $b$   
to M, and draw the line Q M, so shall the angle  
 $b$  Q M, be the course.

3dly, Lay off the proper difference of latitude  
from Q to  $g$ , and draw the parallel  $g$   $k$ , so shall Q  $k$   
be the distance.

We have now projected all the various cases of  
plain, and *Mercator's* sailing, whereby the errors  
of the plain chart are conspicuous; it is also evident,  
they are corrected in *Mercator's* chart; so that both  
charts are absolutely necessary; and strictly speak-  
ing, plain and *Mercator's* sailing cannot be sepa-  
rated, the one being imperfect without the other;  
for, as the difference of longitude cannot be found  
on the plain, so the distance cannot be found on  
*Mercator's* chart, unless the triangle be first projec-  
ted by the principles of the plain chart. We shall  
now calculate for the difference of longitude by  
three different proportions, to shew their agreement  
with the projections.

CASE I. The proper difference of latitude is 61,  
meridional difference of lat. 95.9, departure 169.5  
and the course 70 deg. 19 min. the middle lati-  
tude 50 deg. 30 min.

As radius	10.
Tangent 70 deg. 19 min.	10. 446452
Meridian difference of latitude 95.9	1. 981819
Difference of longitude	268.1 2. 428271

Proper

Proper difference of latitude 61	<u>1.782672</u>
Departure (See p. 80.) 169.5	<u>2.229124</u>
Merid. difference of lat. 95.9	<u>1.981819</u>
	<u>4.210943</u>
Difference of longitude 268.1	<u>2.428271</u>
As co-sine of middle latitude 50 deg. 30 min.	<u>9.803510</u>
Is to radius	<u>10.000000</u>
So is the departure 169.5	<u>2.229124</u>
To the difference of longitude 266.5	<u>2.425614</u>

## CASE VII. As meridional difference

of latitude 1175	<u>3.07004</u>
Is to the difference of long. 704	<u>12.84757</u>
So is radius	<u>10.</u>
To the tangent of the course N 30 deg. 56 min. W.	<u>9.77753</u>
As co-sine course 30 deg. 56 min.	<u>9.93337</u>
Is to radius	<u>10.</u>
So is proper difference of latitude 660	<u>12.81954</u>
To distance 769.5 miles	<u>2.88617</u>

And by using the same operations for the rest, we shall find the latitudes and longitudes to be as follows.

Lat.	N.	Lon. W.	Lat.	N.	Lon. W.
I	51 01	4 28	N	59 59	4 13
K	54 21	8 09	O	55 39	0 02
L	59 23	9 59	P	51 37	1 45
M	61 00	9 59	Q	50 00	1 45

Having now explained all the varieties of what is commonly called plain and *Mercator's* sailing, we shall, in the next place, consider parallel, and mid-

middle latitude, sailing, which we shall likewise unite; for they both suppose the distance betwixt the meridians of two places to be given, either in a parallel of latitude, or in the equator; the proportions will be the same, as to find how many miles will make a degree in any parallel of latitude; or, having the distance betwixt two places in the same parallel of latitude to find their difference of longitude. This will admit only of two cases; and though they have been sufficiently illustrated, in the projection of the sphere, and also on *Mercator's* chart, we shall now treat of parallel and middle latitude sailing as distinct parts of navigation; and explain the principles of each, both geometrically and by calculation.

**CASE 1.** A ship at Q, in the parallel of 50 degrees, sails west to A, and has made 1 deg. 46 min. difference of longitude; required the departure made? or, which is the same, the distance sailed from Q to A? *Plate II. Fig. 10.*

Here the ship makes no difference of latitude, so the departure and distance are the same things. Therefore to construct the triangle,

1st. Draw a line parallel to the margin, as in the first case of plain sailing, and make an angle of 40 degrees, that is the complement of the latitude at the point Q.

2dly, Lay off 106 miles, the difference of longitude from Q to A; from A let fall a perpendicular to cut the line parallel to the margin in B; so shall AB, be the departure, or distance sailed.

**CASE 2.** A ship in 50 degrees latitude, sails west 68.3 miles; required the difference of longitude?

This is only the reverse of the preceding, for having drawn the line A B, make a right angle at B; and lay off the departure 68.3, from B to A; then

**Sect. II. Parallel, and Middle Latitude Sailing. 93**

then make the angle at A, 50 degrees, that is the latitude, so shall the angle at Q be the complement of the latitude, and Q A the difference of longitude.

*Example in Middle Latitude.*

A ship in 39 degrees north latitude, and 6 deg. 48 min. west longitude, sails N W by W. 216 miles; required the latitude and longitude come to?

1st Construct the triangle as in the first case of plain sailing, so shall A B be the difference of latitude, B C, the departure, and C the latitude come to? *Plate 2. fig. 11.*

2dly, Add the latitude sailed from to the latitude come to; the half of their sum will be the middle latitude, and it will be nearly the same thing as if she had sailed west, till she had made the whole departure in that parallel; therefore, as in the preceding, make an angle at C of the same number of degrees with the middle latitude, so shall C D be the difference of latitude; and if the meridional difference of latitude be set off from A to F, as in the first case of *Mercator*, we shall find F G the difference of longitude, equal to C D. The course is 56 deg. 15 min. and the proportions as follows,

R : sine of 56 deg. 15 min. :: distance 216 : departure 180. R : cosine 56 deg. 15 min. :: distance 216 : difference of latitude 120.

Latitude sailed from	39° 00'
Difference of latitude	2 00
Latitude come	41 00
Latitude come to	41° 00'
Latitude sailed from	39
Sum	80

As

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As co-sine middle latitude 40° 00'	9.884254
Is to radius	10.
So is departure 180 miles	<u>12.255272</u>
To difference of longitude 235	2.371018
Latitude.	Meridional parts.
41	2701.6
39	<u>2545.0</u>
Meridional difference of latitude	156.6
As proper difference of latitude 120	<u>2.079181</u>
Is to departure 180	2.255272
So is meridian difference of lat. 156.6	<u>2.194792</u>
	4.450064
To difference of longitude	234.9 2.370883

Having thus gone through the particulars preparatory to Navigation, it remains now to shew, how, by a proper application of these, the mariner may attain what is intended by this art, *viz.* to direct a ship's course to any proposed port.

The first thing necessary to be known, is, the latitude and longitude both of the port, the ship is bound to, and of the place the ship is in; then the course and distance is found by Case 7th of *Mercator*: And, being provided with a compass, he may steer the direct course, if the wind will permit, and the log-line will give the distance he is to sail on that course. The latitudes and longitudes of places may be had from tables made from actual observations, or from charts wherein all the coasts are laid down; and here we think it will be proper to shew how to find the courses and distances on the charts.

S E C T. III.

*Of measuring the Distances of Places, and finding the Course from one to another, upon the Sea-Charts.*

Although on *Mercator's* chart, the degrees of the meridian have the same proportion to those of the parallels that they have upon the globe, yet the distances cannot be measured as they are laid down on the chart: For instance, if one place be in 59 deg. 30 min. and another in 60 deg. 30 min. both N, or both S. latitude, and on the same meridian, it is plain their distance on the globe would only be 60 miles, whereas on the chart it is 120 miles; but on all *Mercator's* charts there are directions how to find the true distances, which will admit of four cases.

**CASE I.** When the two places are upon the equator, the distances are true, for here the degrees are 60 miles each, and the equator is divided into degrees; and where the spaces will admit these are subdivided into minutes, which serves for a scale for the chart.

**CASE 2.** When the two places are on the same meridian, find the latitude of each, the difference of latitude converted into miles, multiplying the degrees by 60, gives their true distance.

**CASE 3.** When the places are in different latitudes and longitudes as I K, K L, &c. first find the difference of latitude as before, and convert it into miles, which call the proper difference of latitude, and take it off the equator.

**2dly.** Lay a ruler over both places, and take the proper difference of latitude with a pair of compasses; put one foot to the edge of the ruler and move it along the edge till the other turned about just touches any *E* and *W* line on the chart; then stop the

the foot at the edge of the ruler, and open the other foot to the point where the edge of the ruler cuts the parallel; this measured on the equator gives the true distance; that is to say, that which can be sailed by the compass. This is evidently the same as projecting the triangle on *Mercator's chart*.

**CASE 4.** When the two places are in the same latitude as A and Q in the parallel of 50, or b and M in the parallel of 61; here is given their difference of longitude; so their difference may easily be had by calculation, as in the preceding section; but if it be required to measure it on the chart, first take, with a pair of compasses, one degree of the meridian at the parallel, that is from 49 deg. 30 min. to 50 deg. 30 min. if the places be in the parallel of 50 deg. or from 60 deg. 30 min. to 61 deg. 30 min. if the places be in the parallel of 61. Then count how many times that extent of the compasses, which is a degree of the meridian at that parallel, will be contained betwixt the two places, and this multiplied by 60 gives the distance in miles, and if it does not measure it exactly, that is to say, if the distance betwixt the places in the parallel be any number of whole degrees of the meridian, and something more, take that overplus in the compasses and apply it to the graduated meridian, so that one foot of the compasses may be as many miles below, as the other is above the parallel, so shall the miles betwixt the feet of the compasses be the odd minutes, and when added to the former converted into miles, will give the true distance. If it be required to a greater exactness, project a triangle as in the 6th Case of *Mercator* for finding the distance from Q to A. This may be done by laying a ruler over one of the places, in such a manner that the angle it makes with the parallel

### SECT. III. Of Measuring Distances on the Chart. 97

parallel, may be the same number of degrees as the complement of the latitude ; as for example, suppose two places  $r$  and  $t$ , both in 56 deg. 15 min. North latitude, and their departure required : As there are several compasses on every chart, the N E by E rhumb line, makes an angle of 56 deg. 15 min. with all the meridians, and of consequence, an angle of 33 deg. 45 min. with all the parallels, which is the complement of the latitude, so it is only laying the ruler over  $r$ , parallel to the N E by E rhumb line, and then placing one foot of the compasses in  $t$ , open the other just to touch the edge of the ruler, which, measured on the equator, will give the true distance, and in this case will reach from  $t$  to  $s$ . Upon the plain chart, there is generally a scale of miles or leagues, by which the distances of places may be measured : And as to the bearings, or the courses from place to place, it is only laying a ruler over the two places, and the rhumb line parallel to the edge will give the course.

The mariner having thus found the course and distance to his intended port, either by the tables, or the chart, in proceeding on his voyage he alters his latitude and longitude, and must therefore find the latitude and longitude the ship is in every day at noon ; and this is performed by the first Case of *Mercator's*, for the course is given by the compass, and the distance by the log-line, but then as a ship may alter her course perhaps every hour, it would be an endless labour to project every single course and distance, or to find the latitude and longitude by calculation. In practice, the various courses and distances are every 24 hours reduced into one single course and distance by the table of difference of latitude and departure ; and this is what may be properly called working a traverse ; for as a ship seldom alters her latitude above two

O degrees

degrees in 24 hours, there will be no occasion to work for the difference of longitude to every single course and distance; it will be sufficient after they are reduced to one course and distance, to find the difference of longitude for the whole 24 hours, which may likewise be done by this table. And as it is by this table the whole practical part of navigation is performed, we shall here shew how it may be constructed.

## S E C T. IV.

*To make the Table of Difference of Latitude and Departure.*

This table gives, by inspection, the difference of latitude and departure to any course from 1 degree to 90, for any distance from 1 mile to 100. Now, if the difference of latitude and departure be calculated for 1 mile, to every degree and quarter point of the compass, we shall have the upper line of all the left hand pages in the table; the rest of the lines are only so many repetitions of the first, which may be found either by addition or multiplication: the second is double the first; the first and second gives the third; the first and third gives the fourth &c. All then that is to be done, is to calculate for the first line, which will be only so many different cases of the first case of plain sailing, the course and distance being always given. Now as the distance is always 1, and the course always given, which suppose 28 deg. we shall find the whole may be performed without any calculation, for if we were to work for it, the operation would be

As radius	10.000000
Is to co-sine of 28 deg.	9.945935
So is distance 1 mile	0.000000
To difference of latitude.	

As

As the logarithm of 1 is 0, it is plain when the terms are placed as above, the sum of the second and third terms, will always be less than the first term, which shews that neither difference of latitude nor departure will be one mile. Let us then suppose the mile to be divided into 10000 equal parts, we may then find how many of these will be in the difference of latitude and departure, by the following operation.

Radius	10.000000
Co-sine 28 deg.	9.945935
Distance 10000	4.000000

Difference of latitude 88.29 3.945935

Now as the first and third terms are always cyphers except the indexes, the fourth term will be the same as the second, except the index: and as 4 is always the index of the third, and 10 that of the first, instead of adding 4 to the index of the second term, and subtracting 10 from their sum, we may subtract 6 from the index of the second term, as in the preceding operation, 6 from 9 remains 3; the same as 4 and 9 is 13; and 10 from 13 remains 3.

Hence the following general rule may be deduced.

1. Look for the sine and co-sine of the course, either in degrees, or points of the compass, in their proper columns, in the table of artificial sines.

2. Subtract 6 from the indexes, and find the natural numbers corresponding to each in the table of logarithms, which will be the difference of latitude and departure, in ten thousand parts of a mile; so if the course be 28 deg. and the distance one mile, the difference of latitude will be 0.8829, and the departure 0.4695; but as the table goes no nearer than tenths of a mile, if it is less than half a tenth it is not regarded; and when more than a half, it is

is accounted one tenth more than is expressed by the first figure; so in the table you will find if the course be 28 deg. and distance 1 mile, the difference of latitude will be .9, and departure .5.

If the course is 1 degree, and distance 1 mile, the sine of the course in the artificials is 8.241855; subtracting 6 from the index, there remains 2, the natural number corresponding to which is 174.5, and this, being less than one tenth part of a mile, it is rejected, and therefore in the table there is no departure; against the co-sine of 1 deg. in the artificials is 9.999934; the proper index after subtracting 6 will be 3, the natural number corresponding to which is 9998 ten thousand parts of a mile, which is in effect, 1 mile as in the table.

Having thus found the differences of latitudes and departures for 1 mile, from 1 degree to 45, we have them likewise without any calculation, from 45 to 90; for if the course be 45 deg. the complement is also 45, which makes the sine and co-sine the same thing, and of consequence the difference of latitude and departure are equal; if the course be above 45, suppose 62, its complement is 28, of which we have the difference of latitude and departure as before; now the co-sine of the course is always the difference of latitude, and the sine of the course is always the departure; hence the differences of latitudes to all under 45, will be the departures to their complements to 90, as in the annexed table.

D. eg.	28 Degrees.		2 $\frac{1}{2}$ points.		49 Degrees.		2 $\frac{1}{2}$ points.	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
1	0.8829	0.4695	0.8819	0.4714	0.8746	0.4848	0.8579	0.5141
2	1.7658	0.9390	1.7638	0.9428	1.7494	0.9699	1.7158	1.0282
3	2.6487	1.4085	2.6457	1.4142	2.6283	1.4584	2.5737	1.5423
100	88.29	46.95	88.19	47.14	87.46	48.48	85.79	51.41
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	62 Degrees.	5 $\frac{1}{2}$ points.	62 Degrees.	5 $\frac{1}{2}$ points.	62 Degrees.	5 $\frac{1}{2}$ points.	62 Degrees.	5 $\frac{1}{2}$ points.

By

Sect. IV. Of the Table of D. Lat. and Dep. 101

By comparing the above with the table of difference of latitude and departure, in the books of Navigation, they will be found to agree exactly. It would be quite unnecessary to calculate for any more, this being sufficient to demonstrate the principles on which the table is constructed.

The table being thus constructed, all the cases of trigonometry, where the hypotenuse does not exceed 100, may be done by inspection, which is the method always used in working a day's work; and because the distances to the several courses, for the most part, seldom exceed 100, the error in 24 hours will not be one tenth part of a mile; but where the distances are larger, they may be taken off in parts. We shall illustrate this by working the six preceding examples, as in the following table.

Courses.	Dist.	N.	S.	E.	W.
W.N.W. $\frac{1}{4}$ W.	180	60.6			169.4
N.W. by N.	240	199.5			133.4
N. by W.	308	302.1			60.1
North.	97	97.			
E $\frac{3}{4}$ S.E. $\frac{1}{4}$ E.	180		60.6	169.4	
S.E. by S.	240		199.5	133.4	
S. by E.	308		302.1	60.1	
South.	97		97.		

Lat. in	Mid.Lat.	Diff. Long.	Long. in
° 1	° 1	° 1	° 1
51 1	50 30	4 27	4 27 W.
54 21	52 41	3 41	8 8
59 23	56 52	1 50	9 58
61 00			9 58
59 59	60 30	5 44	4 14
56 39	58 19	4 14	
51 37	54 8	1 42	1 42 E.
50			

In

very particular in the log-book, and to preserve the operations of every day's work, to which the journal may refer for particulars.

It must be observed, that the course steered is not the true course the ship makes, and therefore the course set down in the traverse table must be corrected by allowing for variation and lee-way.

The variation may be found by taking the sun's amplitude, at his apparent rising or setting, or by his azimuth, when above the horizon, as shall be explained in the next chapter; but when no observation can be made, recourse may be had to Mr Mountaine's Variation-chart, which he has constructed with great accuracy from a collection of near a hundred thousand observations.

There is yet no expedient found for discovering the exact lee-way; this must therefore be attained by experience; but it will very much assist the judgment if the ship's wake be set every time the log is hove; which is practised in some ships, and a column on the log-board reserved for that purpose.

After the variation and lee-way are determined, we must make a traverse-table of six columns; in the first of which must be set down the several courses corrected, and in the second their corresponding distances; we must then find by the table, the difference of latitude and departure to every course and distance, and set them down in their proper columns.—Now, if the several differences of latitude be of one name, *viz.* all north or all south, their sum will be the whole difference of latitude made the preceding 24 hours; but if they be of different names, the difference of their sums will be the whole difference of latitude. The same is to be understood of the departure.

The

The latitude the ship is in, is found by subtracting the difference of latitude, from the latitude sailed from, when sailing towards the equator; or adding thereto when sailing from it. In like manner the longitude is found by adding, or subtracting, according as we sail from or towards the first meridian; but the difference of longitude must be found by the following rule: as, was before observed,

1. Add the latitude at noon to that of the preceding noon.
2. Take half that sum, which call the *Mid. Lat.*
3. Look for the middle latitude, as if it was a course, amongst the degrees in the table, or the complement of the mid. lat. and find the departure in its proper column,
4. Find the departure in the latitude column, corresponding to the middle latitude, now supposed to be a course, and the distance corresponding thereto will be the difference of longitude.

Having thus found the latitude and longitude the ship is in, the course and distance to any port may be easily found, for the difference of latitude and difference of longitude may be had by subtraction, and the departure may likewise be found by the following method, *viz.*

1. Add the lat. of the ship and port into one sum.
2. Take half that sum for the middle latitude.
3. Look for the middle latitude as if it was a course amongst the degrees.
4. Find the difference of longitude in the distance column corresponding to the middle latitude converted into a course, and the difference of lat. corresponding to that distance, will be the departure; so that this is only the reverse of the rule for finding the difference of longitude.

The following example contains all the varieties in a day's work.

H.	K.	F.	Courses.	Winds.		Lee-Way.
				S. W.	N. E.	
1	6	3				Lizard N. b. E. 6 leg. 49° 57' N. lat. 5° 14' W. long. Fresh gales and clear weather
2	7	4	S. W. b. W.	E. N. E.		
3	7	3		N. E. by N.		
4	7	4		N.		
5	7	4				Cloudy
6	7	4				Wind increases, and the sea rises from the N.W.
7	6	6	N. by W.			Single reef top-sails, and handed small sails
8	6	6	N. W.			Blows hard; in all reefs TS
9	6	3	N. W. by W.	1 $\frac{1}{2}$		Handed TS a great sea with rain
10	5	4	W. S. W.	N. W.	3	
11	4				4	Cloudy, no obser. var. per chart 1 $\frac{1}{2}$ points nearly
12	4					Laid to under a mainail
13	3	4				
14	2	2	W. b. N. of W. b. S.	N. by W.	5	
15	3	4				
16	4					
17	5		The wind shifted to E.	E.	1 $\frac{1}{2}$	Moderate STS, wore ship out all reefs TS
18	3		S. S. E.			Fine clear weather, and brisk gale
9	5					28° 06'
10	6					Zenith distance 19° 54'
11	7					Declination
12	8					Latitude 48° 00'

Course

Course corrected.	Dist. in miles.	N.	S.	E.	W.
S. $\frac{3}{4}$ E.	18		17.8	02.6	
S. W. by S $\frac{1}{4}$ W.	41		32.9		24.4
S. by W. $\frac{3}{4}$ W.	12		11.3		04.0
S. by W. $\frac{1}{4}$ W.	8		07.8		01.9
S. $\frac{1}{4}$ W.	6		06.0		00.3
S. by W. $\frac{1}{4}$ W.	6		05.8		01.5
S.S.E. $\frac{1}{4}$ E.	17		15.4	07.3	
S.E. by S. $\frac{1}{4}$ E.	15		11.1	10.1	32.1
<hr/>					
Course made good.	Dist.		D. lat.	20.0	 Dep.
S. $7^{\circ} 00' W.$	110		108.1		12.1
<hr/>					
Lat. failed from				49°	57'
D. lat. S.				1	48
<hr/>					
Lat. per account at noon				48	09
<hr/>					
Sum of lat.				98	06
$\frac{1}{2}$ Do. is the M. lat.				49	03
<hr/>					
Mid. lat dep. in lat. col.	49	12	in Dist. Col. is the diff. long.	0°	19'
Long. failed from				5	14
<hr/>					
Long. in at noon				5	33
<hr/>					

The ship is supposed to sail from the *Lizard*, and must have sailed S by W to the place she was in at 2 o'clock, because it then bore N by E. The variation is  $1\frac{3}{4}$  points westerly, which must be allowed to

to the left hand, and makes the course corrected, S.  $\frac{3}{4}$  E. 18 miles distant; then steered S. W. by W. till 8, with a large wind, allowing variation, makes S. W. by S.  $\frac{1}{4}$  W. 41 miles; from 8 to 10 she makes  $1\frac{1}{2}$  points leeway, corrected by variation, is S. by W.  $\frac{3}{4}$  W. 12 miles; from 10 to 12 leeway and variation allowed, the course corrected is S. by W.  $\frac{1}{4}$  W. 8 miles; from 12 to 2, 4 points leeway, with the variation is S.  $\frac{1}{4}$  W. 6 miles; from 2 to 7 lay to, the middle point betwixt her falling off and coming up is W. allowing 5 points leeway with the variation makes S. by W.  $\frac{1}{4}$  estimating her drift  $1\frac{1}{2}$  miles per hour is 6 miles; from 6 to 10 allow  $2\frac{1}{2}$  points leeway, but the variation being the contrary way, makes the course corrected S. S. E.  $\frac{1}{4}$  E. 17 miles; from 10 to 12 goes large, so there is no leeway, and allowing variation makes the course S. E. by S.  $\frac{3}{4}$  E. 15 miles. The courses being thus corrected, and the difference of latitude and departure to each being set down in their proper columns, the sum of the westings is 32.1 from which subtracting the sum of the Eastings 20.0 there remains 12.1 the departure West. The sum of the Southings is 108.1 the difference of latitude, and because these are too large for the tables, take their halves, the nearest to which is 54.6 difference of latitude, and 6.7 departure, the course corresponding to which is S. 70 deg. West, distance 35, and this doubled makes 110 as per column.

In order to find the latitude, subtract the difference of latitude made from that sailed from, because the ship is in North latitude, and sailing to the southward, which makes the latitude by account 48 deg. 9 min. To find the difference of longitude, the mid. lat. is 49, as to the odd minutes, they need not be regarded, look therefore for 49 degrees in the table, which will be at the bottom of

the

the book, and in the latitude column corresponding thereto, find 12.1 which is the departure, but 12.5 is the nearest, the distance corresponding to it is 19, which is the difference of longitude, and because the ship is in west longitude, and sailing to the westward, it must be added to the longitude sailed from, which makes the longitude the ship is in 5 deg. 33 min.

Now having the latitude and longitude the ship is in, to find the course and distance to any other port, suppose *Cape Finister*, use the following method as in the operation: 1<sup>st</sup>, Find the difference of latitude and difference of longitude, which converted into leagues is 98 the difference of latitude, and 89 the difference of longitude. 2<sup>d</sup>, Find the middle latitude, which is 45 deg. 33 min. or 46 deg. nearest, this will be found at the bottom of the table. 3<sup>d</sup>, Look for 89 (the difference of longitude) in the distance column; and in the latitude column corresponding to that distance, and 49 deg. you will find 61.8; but in 45 deg. (or which is the same thing 4 points) is 62.9, so we may call the departure 62 miles, and difference of latitude 98, the halves of which are 49 and 31, the nearest to these in the tables is 49.2 difference of latitude, and 30.7 departure, which makes the bearings of *Cape Finister* S. 32 deg. W. distance 58 leagues, which doubled is 116..

Ship	lat.	48° 00' N.	long.	5° 33' W.
<i>Cape Finister</i>		43 06 N.		10 00 W.
Diff. lat.	—	4 54—98 leg.	D. long.	4 27—89 leg.
Sum lat.	91 06	Diff. long.	Dep.	Diff. lat.
116M, lat. 45 33		89	62	98
				116lgs S 320 W.

In

In this day's work, we find an error of 9 miles in the latitude, which is corrected by observation; and tho' it is very probable there may likewise be an error in the longitude, yet, till the longitude can be found by observation at sea, we must take that by account, which, it is presumed, will be safer than correcting by the common rules; for a current setting S. E. or S.W. about half a mile in an hour, would make the latitude observed agree with that by account; and if we happen to mistake the current by allowing S. E. when it is S.W. we make an error of 26 miles; whereas, if we trust to our account, the error will be only 13 miles. We are indeed sure, that the distance by account is too short; but whether that be owing to the line, glass, or current, is very uncertain. If the drift and setting of the current could, by any expedient, be found, every time the log is hove it might be accounted for, as if it were a course and distance, and set down with the other courses and distances in the traverse table.

After working each day's work from the log-book, they may from thence be transferred into the journal, the form of which is hereunto annexed.

*Journal*

*Journal on board his Majesty's Ship MAGNANIME, A. D. 1759.*

Remarks on board his Majesty's Ship *Magnanime*.

At 2 PM the Lizard bore N.b.E. 6 leagues, we had then a fresh gale and clear, at 8 blowing hard, in all reefs, and at 2 AM it blew a mere storm, so that we were obliged to lay to till 6. The remaining part mode-

## C H A P VI.

*Of finding the Latitude and Variation of the Compass  
by Cœlestial Observation.*

WE have, in the preceding chapter, shewn how to find the latitude and longitude the ship is in every day at noon, and likewise, from thence, to find the course to any port, which is what was at first proposed to be performed by the whole art of navigation; but, there are two difficulties which seem to be insurmountable; and yet, if not removed, will destroy the very foundation of the whole structure. The first is, that the circles from whence the latitudes and longitudes are accounted are only imaginary on the surface of the earth, and therefore invisible; how then is it possible to know, when we are upon the equator, or first meridian, or what distance we are from either?

In answer to this, it was observed in geography, that all the circles which are drawn on the terrestrial globe, are likewise drawn on the coelestial. Now the heavens are visible, where we actually see the sun and stars every 24 hours describing circles. The sun, when in the equinoctial, which happens twice every year, by the diurnal revolution, either of the earth or sun, describes the equator in the heavens; and altho' all astronomers allow the motion to be in the earth, because they cannot by any machinery exhibit the various appearances of all the planets, supposing the sun to move round the earth; yet, as we consider only the different appearances that the sun, or fixed stars, make to the inhabitants of this earth, we may very safely allow the sun to move round the earth as it actually appears to our senses, which may be exhibited by a simple machinery of pastboard.

S E C T.

S E C T. I.

*To find the Latitude by Observation.*

Let  $ZP\Omega QN SH\mathcal{A}$ , be a meridian in the heavens,  $P$  the north and  $S$  the south pole. Let  $ABCDEFGK$ , be the earth. To an inhabitant any where upon the earth's surface suppose at  $A$ ; (the whole heavens together with) the sun and stars would appear to move round every 24 hours; the point  $P$  seemingly immovable, always appearing to him at the same elevation, which must therefore be the celestial pole, and  $PS$  the axis passing through the center of the earth; then will  $B$  be the north, and  $F$  the south pole of the earth;  $KD$  the equator, on the earth, which, if produced to the heavens, would be  $\mathcal{A}Q$ , the equinoctial. The arch  $KA$ , on the earth, would be the latitude of  $A$ ; and  $Z$  the zenith: It is evident the arches  $KA$  and  $Z\mathcal{A}$  contain the same number of degrees; and as the spectator moves either towards  $K$  or  $B$ , he will have a new point in the heavens for his zenith, just as many degrees distant from  $Z$  as he has moved from  $A$ . Now let us suppose the sun in the equinoctial, or a star visible in the pole. We may then, by an instrument, take the degrees the sun is distant from the zenith, or the degrees the pole is elevated above the horizon, and either of these will give us the latitude, for these two arches are equal. *Plate II Fig. 13.*

Now, though there is no star in the pole, neither is the sun in the equinoctial but twice in the year, yet as the sun's declination, and that of several stars is known, the latitude may be had by the following rules; but it is to be observed, the sun's meridian altitude, or zenith distance, must first be obtained by a quadrant, or some other instrument, and because

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the sun and stars in some latitudes perform their diurnal revolutions above the horizon, whereby they will be twice on the same meridian in 24 hours, once above, and once below the pole, it must be known whether they are at their greatest or least altitude; and likewise, whether we are in south or north latitude at the time of observation.

CASE I. Latitude and declination both north, and zenith distance, south; or both south, and zenith distance north, add the declination to the zenith distance, the sum is the latitude.

EXAMPLE 1.

$$\begin{array}{r} \text{Decl. } 22^{\circ} 18' \text{ N.} \\ \text{Z. dist. } 26^{\circ} 20' \text{ S.} \\ \hline \text{Lat. } 48^{\circ} 38' \text{ N.} \end{array}$$

EXAMPLE 2.

$$\begin{array}{r} \text{Decl. } 2^{\circ} 16' \text{ N.} \\ \text{Z. Dist. } 1^{\circ} 12' \text{ S.} \\ \hline \text{Lat. } 3^{\circ} 28' \text{ N.} \end{array}$$

CASE 2. Declination and zenith distance both of one name, that is both north, or both south, and the latitude of a contrary name, subtract the zenith distance from the declination, the remainder is the latitude.

EXAMPLE 1.

$$\begin{array}{r} \text{Z. Dist. } 70^{\circ} 56' \text{ S.} \\ \text{Declin. } 22^{\circ} 18' \text{ S.} \\ \hline 48^{\circ} 38' \text{ N.} \end{array}$$

EXAMPLE 2.

$$\begin{array}{r} \text{Z. Dist. } 5^{\circ} 44' \text{ S.} \\ \text{Declin. } 2^{\circ} 16' \text{ S.} \\ \hline 3^{\circ} 28' \text{ N.} \end{array}$$

CASE 3. Latitude, declination, and zenith distance, all three of one name, this can only happen to those who are within the tropics; and the latitude is found by subtracting the zenith distance from the declination.

*Example.* On the 20th of May, 1759, the sun's meridian altitude, 74 deg. 26 min. and at the same time to the northward of me; required the latitude?

	90	Declin. by the table 20 0 N
Merid. Alt.	74 26	Zenith dist. 15 34 N
Z. Dist	15 34	Latitude 4 26 N

In all the preceding examples the sun is supposed to be above the pole; but when the sun or star is below the pole, to the altitude, add the complement of the declination; and their sum will be the latitude.

*Example.* Observing the upper star of the two last in the square of the *Great Bear* to be 18 deg. 34 min. above the horizon; required the latitude? I find by the table, the star's declination is 58 deg. 34 min.; the complement is 31° 26' N

Altd. 18 34

Lat. 50 00 N

The latitude and declination may likewise be had, by observing the greatest and least altitude of any star which makes an entire revolution above the horizon in 24 hours; as in the following example; observing the aforesaid star's zenith distance to be 8 deg. 34 min. the complement 81 deg. 26 min. is the altitude.

Least altitude	18° 34'	Greatest alt.	81 26
Greatest	81 26	Least	18 34
Sum	100 00	Diff.	62 52

$\frac{1}{2}$  sum is the lat. 50 00  $\frac{1}{2}$  is com. dec. 31 26

*Demonstration.* Draw the circle, and quarter it as before; and because H is the south point of the horizon, and the sun to the southward of the zenith, in the 1st and second case, lay off 26 deg. 20 min. from Z to R, which will be the sun's place in the heavens at the time of observation, and as the sun has north declination, it is certain the equinoctial must be to the southward of the sun, therefore lay off 22 deg. 18 min. the declination from

from  $R$  to  $\text{AE}$ ; so shall  $Z \text{AE}$ , be the latitude, the sum of  $Z R$  and  $R \text{AE}$ ; again, in the 2d case, 70 deg. 56 min. is the zenith distance which must be laid off from  $Z$  to  $U$ , and because the declination is south, the sun must be to the northward of  $U$ , therefore laying off 22 deg. 18 min. from  $U$ , we shall have  $\text{AE} Z$  the latitude, which is the difference betwixt the zenith distance and declination. In the 3d example,  $Z r$  is the zenith distance, and  $r \alpha$  the declination, so shall  $\text{AE} Z$  be the latitude.

When the star makes an entire revolution above the horizon, the zenith distance, at the time it is above the pole, is  $Z 4$  and the altitude, when below the pole, is  $O 5$ , and drawing the dotted line which is bisected by the radius  $C p$ , we shall have  $O p$  the lat. and  $p 4$ , or  $p 5$ , the complement of the declination, and it is plain this is half the arch  $4 p 5$  the difference betwixt the greatest and least altitude. That the elevation of the pole above the horizon is equal to the latitude, may be thus proved; the arch  $\text{AE} Z$ , and  $Z P$ , make 90 degrees, but the arch  $Z P$  and  $P O$  are 90 degrees, therefore  $\text{AE} Z$ , the latitude, is equal to  $P O$  the elevation of the pole. *Plate 5 Fig 1<sup>st</sup>.*

## S E C T. II.

### *Of the Right Ascension of the Sun and Stars.*

The astronomers have calculated tables of the declination of the sun, and of all the remarkable stars; but if we do not know at what hour the stars come to the meridian, we shall be at a loss when to begin to observe; to remedy this, the right ascension of the sun and of all these stars are likewise collected into a table; so the hour that any star comes upon the meridian may easily be found.

In

In order to give us a clear idea of the right ascension of the sun or of the stars, it will be necessary to observe, that their apparent motion is from East to West; the stars rise and set all the year, at least nearly, in the same points of the horizon, so their declinations may be allowed to continue the same, and of consequence may be supposed fixed in the heavens, on each side of the equinoctial; and the whole heavens, together with the equinoctial, sun and stars, performing an entire revolution in 24 hours. The stars will return to the meridian in the same space of time they performed their revolution the preceding 24 hours; but this is not the case with the sun, for he has an apparent annual as well as diurnal motion; so that he cannot, like the stars, be a fixed point in the heavens, else he would like them rise and set always in the same point of the horizon; and that it is not so, is evident to every man's observation, without the assistance of any instruments.

The sun's annual motion is from West to East, in the ecliptick, which is a great circle in the heavens, intersecting the equinoctial in two opposite points, at an angle of 23 deg. 29 min. These points astronomers call *Aries* and *Libra*; and as the sun is always in the ecliptick, and performs a revolution in it once a year; he will be twice in the equinoctial every year. Let us then suppose him in *Aries* the 21st of *March*, and at the same time on our meridian.

Now before the heavens has made one entire revolution, the sun has moved near a whole degree eastward in the ecliptick, so that when *Aries* comes next day to the meridian, the heavens must turn near a whole degree more to the westward before the sun comes to our meridian: Hence the point of the equinoctial that comes to the meridian with the sun,

sun, will be near a whole degree further from *Aries*, than that point of the equinoctial which was on the meridian with the sun the preceding day, and so the sun will be continually moving further from *Aries*, till, having performed his annual revolution, he again returns to the meridian with *Aries*.

It will be easy now to conceive what is meant by the sun or star's right ascension; for it is the degrees and minutes of the equinoctial intercepted betwixt the first point of *Aries*, (or rather the point where the ecliptick intersects the equinoctial) and a meridian passing through the sun or star.

The right ascension of the sun and stars in the tables is given in time, allowing 15 deg. to one hour, which makes one degree of the equinoctial four minutes of time, and as the sun's right ascension, is given, when it is on the meridian at noon, it will be easy to find the hour when any star comes on the meridian: For supposing the star's ascension 15 degrees, or one hour, and the sun's ascension two hours, it is plain the star will come to the meridian one hour before the sun that is at 11 hours after midnight, but if the star's ascension be 14 hours, and the sun's 13, the star will come to the meridian one hour after the sun, that is at one hour after noon; hence

*To find the Time of any Star's coming to the Meridian.*

*Rule.* Subtract the sun's right ascension from that of the star, or if the star's right ascension be less than the sun's add 24 thereto, and the remainder is the time of the star's coming to the meridian from noon; if the remainder exceeds 12, subtract 12 from it, the last remainder is the time from midnight.

*Exam-*

**Sect. II. Of the Ascension of the Sun or Stars. 117**

*Example 1.* What time will *Fomelbault* come to the meridian the 21st of *October*.

*Examp. 2.* What time will the *Bull's Eye* come to the meridian the 26th of *October*.

H. M.

From stars ascen.	22	40
Sub. Suns.	13	44
Hour from noon	8	56

H. M.

To stars ascen.	4	21
Add	24	00
	28	21
Suns ascen.	14	03
	14	18
Subtract	12	
Hour from mid.	2	18

Having thus found at what hour the star comes to the meridian, its altitude may be taken, and the latitude found as before directed, when the observation is by the sun.

As it frequently happens that the heavens are some times obscured, it will be necessary to know what stars come upon the meridian when an observation may be had, for which take the following rule.

*To find what Star will come upon the Meridian at any proposed Hour of the Night.*

*Rule.* To the sun's right ascension add the time from noon at which the stars coming to the meridian is required, their sum is the right ascension of the star that comes upon the meridian about that time, and this being found in the table, the star corresponding thereto will be that required, and proper to observe by.

*Ex-*

*Example.* What star will come upon the meridian the 7th of April about 8 at night?

	H.	M.
Sun's ascen.	1	4
Time from noon	8	0
Stars ascen.	9	4

The nearest in the table is 9 h. 14 m. the ascension of *Hydras* heart, which therefore comes upon the meridian at 8 h. 10 min.

### S E C T. III.

#### *Of the Variation of the Compass.*

The compass is the only instrument the mariner has to direct his course; but if the magnetick needle does not point out the true meridian, which is frequently the case, he will be led into a very great error if the variation is not known.

Now if the sun or star be on the meridian when the altitude is not above 15 degrees, the bearings may be taken by an azimuth compass, and the variation thereby discovered, this can happen by the sun only to those who live within the polar circles; we shall therefore shew how it may be found by the sun's amplitude and azimuth.

When the sun is in the equinoctial, it rises in the East, and sets in the West, and then it has no amplitude; when it has North declination, it rises and sets to the Northward, but when South declination, it rises and sets to the Southward of the East and West, and the degrees of the horizon intercepted betwixt the East, and the sun when rising, or the West when setting, is the sun's amplitude.

### P R O B L E M I.

The latitude and sun's declination given; to find his amplitude.

*Exam-*

*Example.* Latitude 48 deg. 38 min. declination 22 deg. 18 min; required the sun's amplitude? Or, which is the same thing, course 48 deg. 38 min. difference of latitude 3795; required the distance?

As co-sine lat. 48° 38'	<u>9.82012</u>
Is to radius	10.
So is sine declin. 22° 18'	<u>9.57916</u>
To sine ampl. 35° 3'	<u>9.75904</u>
As co-sine course 48° 38'	<u>9.82012</u>
Is to radius	10.
So is diff. of lat. 3795	<u>3.57921</u>
To dist. 5743	<u>3.75909</u>

*Demonstration.* R T, the parallel of declination, if North, cuts H O, the horizon, in X, so shall C X be the amplitude equal to C Y, when the declination is South. *Plate V. Fig. 1.*

In the right angled triangle, C f X, equal C o Y; the angle at C is 48 deg. 38 min. that being the latitude, of which the angle at X is the complement. Now, if we call the angle f C X the course, and C f the difference of latitude, C X will be the distance, but C f is the natural right sine of the declination, 3795; and, as we observed before, we shall have no occasion to use the natural sine, for it is plain, by the preceding operation, we take its logarithm, which may be had in the artificials, so the proportion will be, as above, co-sine latitude, : R :: fine declin : fine ampl.

In this example, the true amplitude is 35 deg. 3 min. but if by observation it be found 16 deg. 3 min. Northerly or Southerly, according as the true is, their difference 19 deg. is the variation; but if either the true or magnetick be N. and the other S. as supposing the true 6 deg. 14 min. South, and the magnetick 5 deg. 1 min. North, the variation

is 11 deg. 15 min.; after the variation is found, the course may be corrected; for the true course will be to the left of that steered when the variation is Westerly, but to the right when Easterly, and to know whether the variation is Easterly or Westerly, take this general rule.

When you are looking to the sun at the time of observation, if the magnetick be to the right of the true, the variation is Westerly; but if to the left, Easterly; thus, in the preceding example, if the amplitude was taken at sun rising, the true would be E. 35 deg. 3 min. N. and the magnetick being E. 16 deg. 3 min. the variation is 19 deg. Westerly, and a ship then steering North, would make the true course N. 19 deg. W. If the observation was at sun setting, the true is W. 35 deg. 3 min. N. and magnetick W. 16 deg. 3 min. the variation is Easterly, and then the true course would be N. 19 deg. E.; hence, if the variation is Westerly, it must be allowed to the left, and if Easterly, to the right of the course steered.

## P R O B L E M II.

The latitude of the place, the sun's declination and altitude given, to find his azimuth. This will admit of three different cases, which may be solved by one general proportion; the third term of which must be obtained by a previous preparation.

As the comp. of the lat. and of the altitude are always given, we have the sum or difference of these two arches, by addition or subtraction, and of consequence the natural sine of their sum, or of their difference, and also the nat. sine of their declination may be had in the table. (See Chap. III. Sect. 2.) and with these we are to operate as the circumstances require.

CASE

CASE 1. Latitude and declination both North or both South, and the observation taken in the forenoon after the sun has past the prime vertical, that is, the azimuth circle passing through the East point of the horizon, or in the afternoon before the sun comes to the West; or, which is the same thing, when the sun is betwixt the mid day meridian and prime vertical.

Here we must work by the nat. sine of the difference of the two arches, *viz.* the altitude and complement of the latitude; which is to be added to the natural sine of the declination, when the comp. of the latitude exceeds the altitude: but if the altitude be greatest, the nat. sine of their difference is to be subtracted from that of the declination, and the sum or remainder will be the third term of the proportion.

CASE 2. Latitude and declination both North, or both South, and the sun betwixt the prime vertical and midnight meridian.

Add the altitude to the complement of the latitude, and find the nat. sine of the sum of these two arches, from which subtract the nat. sine of the declination, and the remainder will be the third term of the proportion.

CASE 3. Latitude and declination, one North and the other South. In this case the sun is always betwixt the mid-day meridian and prime vertical, and the comp. of the latitude always exceeds the altitude, and when the difference of these arches is equal to the declination, the sun is on the meridian. Here then subtract the natural sine of the declination of the arches, from the nat. sine of the difference of the arches, and the remainder will be the third term of the proportion. *Note.* If the comp. of the latitude be equal to the altitude, the artificial sine of the declination will be the third term of the proportion:

tion: the general proportion is as co-sine of the altitude is to the secant of the latitude; so is the third term, found as before directed, to the versed sine of the azimuth.

It is presumed, that the variation may be obtained to less than  $\frac{1}{4}$  of a point, by Mr Mountaine's variation chart; so the observer may know whether the sun is betwixt the mid-day or mid-night meridian and the prime vertical, even by the steering-compass; however, if the result of the operation should exceed the radius, from the natural number of that logarithm subtract the radius, which is always 10000; the logarithm of the remainder will be the artificial co-sine of the azimuth, which must be taken from the midnight meridian, if we use the nat. sine of the difference of the arches; but if we work by the nat. sine of the sum of the arches, the azimuth must be taken from the mid-day meridian. It must be observed, if the third term consists of four figures, 9 must be the index of the logarithm; if three figures, 8 must be the index; if two figures, 7; and when it is only one figure, 6 must be the index of the logarithm. Hence we may deduce the following general rule: To the logarithm of the third term, with the proper index prefixed, add the artificial secant of the latitude, and from the sum subtract the artificial co-sine of the altitude, the remainder will be the logarithm of the versed sine of the azimuth. If we have not a table of versed sines, we may subtract 10000 from the natural number of its logarithm, and the logarithm of the remainder will be the artificial co-sine of the azimuth. We shall illustrate the whole by an example in each case.

*Example 1.* Latitude 48 deg. 38 min. North; declination 22 deg. 18 min. North; altitude 41 deg. 22 min. equal to the comp. of the latitude; therefore co-sine

Sect. III. *Of the Sun's Azimuth.*

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co-sine altitude : secant latitude :: sine declination :  
versine azimuth.

To third term sine declination	$22^{\circ} 18'$	9.57916
add secant latitude	48 38	10.17988

Sum		19.75904
Co-sine altitude	$41^{\circ} 22'$ from south.	9.87535
7651 N. versine of azimuth	76 24	9.88369
2349 N. co-sine of azimuth	76 24	9.37088

*Example 2.* Altitude 15 deg. latitude and de-  
clination as before.

Preparation,

To complem. lat.	$41^{\circ} 22'$
add altitude	15 00

Sum	56 22 N. fine 8326
Declination	22 18 N. fine 3794

Difference of the sines	4532	9.65629
Secant latitude	$48^{\circ} 38'$	10.17988

Co-sine altitude	15 deg.	Sum	19.83617
			9.98494

7100 N. ver-sine of azim. from N.	$73^{\circ} 8'$	9.85123
2900 N. co-sine of azim. from N.	73 8	9.46240

Because it may be doubtful on which side of the prime vertical the sun may be, at the time of obser-  
vation, we shall work this example by the nat. sine  
of the difference of the arches,

Complement

Comp. lat.	41° 22'			
Altitude	15 00			
Difference	26 22 N.S. 4441			
Declination	22 18 N.S. 3794			
Sum of N. sines		8235	9.91566	
Secant latitude	48° 38'		10.17988	
Co-sine altitude			20.09554	
			9.98494	
12900 is N. number				10.11060
10000				
2900 N. co-sine as before			9.46240	

*Example 3.* Latitude 48 deg. 38 min. North; Declination 22 deg. 18 min. South; altitude 15 deg. Required the sun's azimuth?

In this, and in all such cases, the sun is betwixt the prime vertical and the mid-day meridian, all the time it is above the horizon; and as the altitude can never exceed the complement of the latitude, we may use the difference of the sines.

Comp. latitude	41° 22'			
Altitude	15 00			
Difference	26 22 N.S. 4441			
Declination	22 18 N.S. 3794			
		647	8.81090	
Secant latitude	48 38		10.17988	
				18.99078
Co-sine altitude	15 deg.		9.98494	
1014 N. ver-sine of 26 deg. 2 min.			9.00584	
8986 Comp.	26 2		9.95356	

After finding the true azimuth, the magnetick may be had by observation, and the course corrected as before by the amplitude.

Having now shewn how to find the latitude and variation of the compass by coelestial observation, our next business shall be to consider how far such observations may assist us in determining the longitude at sea: In order to which, it is necessary to know the exact time even to minutes and seconds of an hour, both at the place the ship is in, and likewise of some other place, whose longitude is known; and that at the very instant of time the observation is taken: We shall therefore shew how to find the hour at sea.

### PROBLEM III.

Latitude, altitude and declination given to find the hour; in this we shall make two different cases, which may be solved by one proportion; but here, as in the azimuths, the third term must be found by a previous preparation, which we shall illustrate by the same examples taken before for the azimuths.

CASE 1. When the observation is taken before six in the morning or after six at night, subtract the declination from the comp. of the latitude, add to the nat. sine of the remaining arch, and the nat. sine of the altitude, and their sum will be the third term; this can only happen when the latitude and declination are both North or both South, as in the 2d example.

CASE 2. When the observation is taken after six in the morning, or before six at night, as in the 1st and 3d examples, subtract the nat. sine of the present altitude, from the nat. sine of the meridian altitude for that day, and the remainder will be the third term; if it be doubtful whether it is past six or not at the time of observation, we may

use

use either of these ; and if the result should exceed the radius, subtract the radius from it, as before for the azimuth, and the logarithm of the remainder will be the artificial sine of the hour before six, if we work by the difference of the nat. sines of the present and meridian altitude ; but if we work by the sum of the sines as in the preceding case, it will be past six. After the third term is found, use the following proportion ; co-sine declin : sect. lat :: third term : ver-sine of the hour. Hence the following general rule.

To the logarithm of third term, add the artificial sect. of the latitude, and from the sum subtract the artificial co-sine of the declin. and the remainder will be the logarithm of the versed sine of the hour. *Plate V. Fig. 1.*

## EXAMPLE I.

H  $\Delta E$  comp. lat.  $41^{\circ} 22' N$

$\Delta E R$  declin.  $22 18 N$

H R mer. alt.  $63 40 N. S.$  8962

H  $\Delta E$  pre. alt.  $41 22 N. S.$  6609

Diff. sines is the third term  $2353$   $9.37162$   
Secant lat.  $48$  deg.  $38$  min.  $10.17988$

Sum  $19.55150$

Co-sine declin.  $22$  deg.  $18$  min.  $9.96624$

N. ver-sine of  $52^{\circ} 02'$  is  $3848$   $9.58526$

Comp. is  $37 58 N. S.$  6152  $9.78902$

$52$  deg.  $2$  min. is before  $12$   $3$  h.  $28$  m.  $8$  s.

$37$  deg.  $58$  min. is after six  $2$  h.  $31$  m.  $52$  s.

So the hour at the time of observation is  $8$  h.  $31$  m.  $52$  s.

## EXAMPLE

## EXAMPLE 2.

Comp. lat.  $41^{\circ} 22' N.$ Declin.  $22^{\circ} 18' N.$ Rem.  $19^{\circ} 04' N. S. 3267$ Prev. alt.  $15^{\circ} 00' N. S. 2588$ Sum of the sines  $5855$   $9.76752$ Sect. lat.  $48$  deg.  $38$  min.  $10.17988$ Sum  $19.94740$ Co-sine declin.  $22$  deg.  $18$  min.  $9.96624$ N. ver-sine of  $87$  deg.  $34$  min. is  $9576$   $9.98116$ Comp. is  $2.26$  N. S.  $424$   $8.62737$ Merid  $2$  deg.  $26$  min. before six  $0$  h.  $9$  m.  $44$  s.8 h.  $22$  m. past midnight  $5$  h.  $50$  m.  $16$  s. $57^{\circ} 34'$   $6$  h.  $0$  min.  $0$  s.

## EXAMPLE 3.

Here the meridian altitude is the same with the remainder in the preceding.

Merid. alt.  $19^{\circ} 4' N. S. 3267$ Prev. alt.  $15^{\circ} 00' N. 2588$ Diff. of sines  $679$ Co-sine declin.  $22^{\circ} 18'$   $9.96624$ Sect. lat.  $48^{\circ} 38'$   $10.17988$ Diff. Sines  $679$   $8.83187$  $19.01175$ N. ver-sine of  $27^{\circ} 15'$  is  $1110$   $9.04551$ Comp. is  $62^{\circ} 45' N. S. 8890$   $9.94890$ 

*Demonstration.* Draw the parallels of altitude  $E K$ , and  $A B$ , also the parallels of declination  $R T$   
 $S$  and

and  $U V$ , so shall  $D$  be the sun's place in the heaven, at the first observation;  $\text{AE} D$  the versed sine of the azimuth to the radius  $\text{AE} G$ , and  $R D$  the versed sine of the hour, to the radius  $R f$ , and drawing an azimuth circle through  $D$ , it will intersect the horizon in  $F$ , so shall  $C F$  be the co-sine of the azimuth, which may be measured on the line of sines, but this may be done without drawing the azimuth circle. Thus, 1st, Draw a radius to meet the parallel of altitude in the circumference, which, in the 1st Example, will be  $C \text{AE}$ , because the complement of the latitude and of the altitude are equal. 2dly, Draw  $D E$  parallel to  $C Z$ , to intersect  $C \text{AE}$  in  $b$ ; so shall  $C b$  be equal to  $C F$  the co-sine, and  $\text{AE} b$  equal to  $H F$ , the versed sine of the azimuth, for as the radius of any parallel of altitude, is to the radius of the horizon, so is the sine of any arch in the parallel of altitude, to the sine of a similar arch in the horizon; (see Chap. III. Sect. II.) now the triangles  $\text{AECG}$ , and  $\text{AE} b D$  being similar  $\text{AE} G$ : (the radius of the parallel)  $\text{AE} C$  :: (equal  $C H$  the radius of the horizon)  $\text{AE} D$ :  $\text{AE} b$  the versed sine of the azimuth; again in the triangle  $\text{AE} k D$ , the angle  $k \text{AE} D$  is equal to the angle  $P C O$ , the given latitude, for  $\text{AE} k$  is parallel to  $P C$ , and  $\text{AE} K$  to  $H O$ ; the side  $\text{AE} D$  is the versed sine of the azimuth to the radius  $\text{AE} G$ ; the side  $\text{AE} k$  is the right sine of the declination, therefore making  $\text{AE} k$  radius,  $\text{AE} D$  will be the secant of the latitude. Now to find  $\text{AE} D$  it will be  $R : \text{sect. Lat} :: \text{AE} k : \text{AE} D$ , the operation is

Radius		10.
Sect. 48 38		10.17988
Sine 22 18		9.57916

$\text{AE} D$  9.75904 *Pl. V. Fig. 1.*

Co-sine alt. 41 d. 22 m.	<u>9.87535</u>
Radius	10.
$\text{AE D}$	<u>19.75904</u>
$\text{AE } b$ versed sine azimuth	9.88369

Here are two proportions, but as in the first the logarithm of the radius is subtracted from the sum of the logarithms, of the sect. of the lat. and of the fine of the declin, the remainder gives the logarithm of  $\text{AE D}$ , now in the second, the logarithm of the radius is again added to that of  $\text{AE D}$ ; so the radius may be taken away in both, and then the operation may be done by one proportion as before.

Again, if we draw a meridian through D, it will cut the equinoctial in  $q$ ; so shall  $C q$  be the fine of the hour from six, and  $\text{AE } q$  the versed sine, which may be found without drawing the meridian after the same manner the azimuth was found; thus, 1st, Draw a radius  $C R$  to meet the parallel of declination in the circumference; 2dly, Draw  $D p$ , parallel to  $P S$ , to intersect  $C R$  in  $p$ ; so shall  $C p$  be equal to  $C q$  and  $R p$  equal to  $\text{AE } q$  the versed sine of the hour, (see the demonstration for the azimuth.) Now  $R e$  is the fine of the meridian altitude,  $a e$  the fine of the present altitude, and  $R a$  their difference. So,  $R : \text{Sect. lat} :: R D = R a \times \text{Sect. lat} \div R$  and co-sine declin :  $R :: R D$  : versed sine hour, and omitting the radius in both, we have, as co-sine declination, is to secant latitude, so is  $R a$ , to the versed sine of the hour, as before.

In the 2d Example,  $g$  is the sun's place in the heavens at the time of observation;  $B g$  is the versed sine of the azimuth to the radius  $n B$  and  $T g$  the versed sine of the hour to the radius  $f T$ ;  $Q O$  is the complement latitude,  $O B$  the altitude, the arch  $Q O B$  is their sum, of which  $B M$  is the N. fine;

sine;  $y M$  the N. sine of the declination and  $B y$  their difference. So, in the triangle  $B g y$ ,  $R : \text{Sect lat} :: B y : B g$ ; and as co-sine altitude is to radius, so is  $B g$  to the versed sine of the azimuth, and omitting the radius, in both, we have the general proportion, as before.

Again,  $Q O$  is the complement latitude,  $Q T$  the declination, and  $T O$  their difference, to which adding  $O B$  the altitude, we have the arch  $T O B$ , of which  $T B$  is the right sine. In the triangle  $T B g$ ,  $R : \text{Sect lat} :: T B : T g$ , the versed sine of the hour, to the radius  $T f$ ; and co-sine declination :  $R :: T g$ ; versed sine hour, and omitting the radius we have the general proportion.

In the 3d Example,  $H \mathcal{A} \mathcal{E}$  is the complement latitude,  $H A$  the altitude, and the arch  $A \mathcal{A} \mathcal{E}$  their difference, of which  $A \mathcal{A}$  is the right sine, from which subtracting the sine of the declination we have  $A x$ , so  $A m$  may be found as before; in like manner in the triangle  $U 1 m$ ,  $U 1$ , is the difference betwixt the sine of the meridian, and the sine of the present altitude, and  $U m$  the versed sine of the hour.

Tho' we may find the azimuth and hour by the preceding problems, yet as the common books of navigation have neither the natural or versed sines, it will be necessary to shew how they may be obtained by the common tables; and as the rules are founded on the stereographick projection of the Sphere, we shall refer for the demonstrations to the authors who have treated of sphericks: but here it will be proper to remark, that the quadrant gives only the apparent altitude; we must therefore find the true by the following method.

1. Add 16 min. for the sun's semidiater, to that on the quadrant, if you observe by the lower edge; but if by the upper, subtract 16 min.

2. Sub-

2. Subtract the refraction, and the allowance for the height of the eye above the horizon from the above sum or remainder. *Note*, if we use a back observation, the allowance for the height of the eye must be added.

Another thing to be observed is, that the declination in the tables is calculated for mid-day, and, therefore, a proportional part of the difference betwixt that, and the declination the preceding day, must be added or subtracted to that in the tables. The latitude must likewise be had at the time of observation, by working for the latitude made by account since last observation. We shall illustrate this by the following example:

*June 3, 1759.* Latitude observed 49 deg. 2 min. apparent altitude 14 deg. 53 min. Required the true?

appar. altitude of sun's	lower edge	14° 53'
	add semidiameter	0 16
	center	15 09
Height of eye for 25 feet	0 6'	Subtract
Refraction	0 3	9
True altitude of the sun's center		15° 0

The declination the second day is 22 deg. 12 m. and the third 22 deg. 20 min. the difference is 8; one quarter of which, 2, being subtracted from 20, because it wants only six hours of noon, makes the declination at the time of observation 22 deg. 18 min. Again, the ship is supposed to have made 24 miles southing since last observation, which makes the present latitude 48 deg. 38 min.

Having thus found the true altitude, latitude, and declination, the azimuth and hour may be found by the following rule:

1. Add the comp. of the latitude, comp. of the altitude, and comp. of the declination into one sum.
2. From

whereby the whole of navigation may be performed by right angled triangles alone, as was at first proposed; and in calculations for the hour it would be proper to work both ways, because this is of great importance in determining the longitude,

There are astronomical tables, calculated to great accuracy, which give the exact time when the eclipses of Jupiter's satellites shall happen at all the royal observatories in Europe. Now, as these eclipses are seen, in different places, at the same instant of time, we need only find the exact time in those places, at the instant the transits are seen, by the preceding problem; for then the difference of time, in hours, minutes, and seconds, being multiplied by 15, would be the difference of longitude, in degrees, &c. betwixt the place of observation, and the observatory, from which the longitude is accounted; and it is, by this means, the longitude of any place is determined on land; but as the satellites cannot be seen by the naked eye, the instant of the transits at sea cannot be obtained, to that nicety, requisite for determining the longitude, unless some expedient be found to prevent the ship's motion from affecting the observer.

How far Mr *Irwin's* Marine Chair may answer this end, our readers may judge by what he has published on that subject. It is certain he made an experiment of it on board his Majesty's ship *Magnanime*, which give great hopes of seeing that important point determined.

#### S E C T. IV.

##### *Description and Use of Davis's and Hadley's Quadrants.*

*Davis's* quadrant consists of two arches, framed and fastened as in *Fig. 2. Plate VI.* Both the arches are

are drawn from the same center; that of the shortest radius contains 60, and that of the longest 30 deg. In order to graduate the arches, from the center H, and with the radius HG, taken from any line of chords, describe a quarter of a circle; so the angle MHG will be 90 degrees, which may be transferred from the dotted arch to those on the instrument; the divisions of the 30 arch beginning in the line HG, and the divisions of the 60 arch in the line HM; so shall the two arches make 90 deg.

It has three vanes, *viz.* the horizon vane, fixed at H; the sight vane, to move on the 30 arch; and the shade vane, to move on the 60 arch; as the circumstances require. The instrument should be in such a position, at the time of observation, that the line KH passing from the eye to the slit in the horizon vane, may be parallel to the horizon, when the shadow of the sun by the shade vane coincides with the upper part of the slit; for then counting the degrees on both arches, their sum will be the sun's zenith distance.

*Demonstration.* Let the line WX be parallel to the horizon; the angles ZHM and MHS, both together, will be the sun's zenith distance; but MHZ is equal to KHG; for if either of these two angles be added to the angle MHK, their sum will be 90 degrees; therefore the sum of the degrees on both the arches will be the sun's zenith distance.

*Hadley's quadrant* is the best instrument for taking altitudes at sea, for the description of which we refer to a pamphlet, published by Mr *Watkins*, optician, at *Charing-Cross*, which explains the use and theory of that noble instrument in a plain familiar manner, intelligible to any ordinary capacity: but as the degrees on the arch are only divided into three equal parts, which are 20 minutes each, it will

be necessary to shew how the altitude may be taken to minutes, and what to allow for the height of the eye above the visible horizon. *Pl. VI. Fig. 3.*

Let *A B* be a part of the arch of *Hadley's* quadrant divided into whole degrees, and these subdivided into three equal parts, which will be 20 min. each. Let *C D*, the chamfered edge of the index, contain 3 deg. which will therefore be 9 divisions of the arch on the quadrant, that is, 180 min. Now if this space on the index be divided into ten equal parts, they will be 18 min. each ; and whereas those on the arch are 20 minutes, it is plain, when the line in the middle of the index unites exactly with any one of the divisions on the arch, so as to form a strait line, it then points out either whole degrees, or subdivisions, which will be either 20 or 40 minutes more, to be added to the degrees ; in that position the other divisions of the index will fall short of those upon the arch ; that marked 2 will be two minutes short of the first division on the arch, next to that which coincides with the line in the middle of the index : the division marked 4 on the index, four minutes short of the second on the arch, &c. so that if the index be moved till 2, 4, 6, 8, or 10 coincide with any of the divisions of the arch, then 2, 4, 6, 8, or 10 min. must be added to those that would have been expressed by the index, had, the line in the middle of it, coincided with any of the divisions of the arch ; and for the same reason, when 12, 14, &c. coincide with any of the divisions of the arch, there must be 12, 14, &c. minutes added. Hence, as it stands in the figure, the index express 2 degrees and 50 minutes.

Now, if instead of 9 divisions of the arch of the quadrant, we take 19, and divide that space on the index into 20 equal parts, they will be 19 minutes each ; by which means the altitudes may be taken

to one minute; and as to the seconds (on the land quadrants) there is a brass nut fitted to slide along the arch, which may be fastened where most convenient; there is also a small plate, like the dial-plate of a watch, fixed to the nut; through the center of this plate goes a screw, with a small index, which moves round the plate, while the index of the quadrant is moved one minute by turning the screw; and the dial-plate being divided into 60 equal parts, its index will point the seconds, and the divisions, on the index of the quadrant, the minutes. *Plate VI.*

*Fig. 3.*

The visible and rational horizon, being always parallel to one another, and the eye of the observer above the visible, as the quadrant stands in the figure, the sight-vane must be moved up to A, before the horizon can be seen, so the zenith distance by the quadrant will exceed the true by the angle A H K.

In order to find this angle, let *m n* be a part of the circumference of the terraqueous globe; R the center; this, in the plate, is in the line Y Z. Now, supposing the eye at H, 40 feet above the surface of the sea, the horizon will then be seen at *m*; H T will be a tangent to the surface of the globe in the point T, and the angle M H T equal to the angle H R T. In the right angled triangle H R T is given R T, the semi-diameter of the earth, which suppose 21123276 feet, and R H will be 40 feet more, that is, 21123316; therefore making T R radius, H R will become the secant of the angle at R, and the proportion will be R T : R H :: R : secant of the angle.

OPERATION.

R T : 21123276	7.324728
R H 21123316	<u>17.324729</u>
Secant of $0^{\circ} 7'$	10.000001

Now

Now because the angle at T is 90 degrees, the angles T H R and T R H both together make 90 degrees; but the angles T H R and T H M both together make 90 deg. therefore the angle T R H is equal to the angle (T H M) A H K. By this operation, the sun's zenith distance, as the quadrant gives it, exceeds the true 7 minutes, which must therefore be subtracted from it; or if we take its altitude, we must add 7 minutes to that on the quadrant.

It must be observed, this is when we use the back observation; but when we take the altitude by the fore observation, as supposing the eye at H, that on the quadrant will be the arch S p, whereas the true is S q; their difference is the arch q H p, 7 minutes, as by the foregoing operation; hence, in the fore observation, which is that generally used by *Hadley's* quadrant, we must subtract the allowance for the height of the eye from that on the quadrant.

*Note.* The proportion that 40 feet has to the diameter of the earth, is so small, that it cannot be represented in the plate, the design of the figure being only to demonstrate the reason of the operation; and an error in any of the figures which we have taken for the semi-diameter of the earth, will not alter the angle half a minute; so the tables of the dip of the horizon may be allowed sufficiently accurate for the mariner, as to what concerns the latitude.

### E R R A T A.

plate	line	read	plate	line	read
✓ 4	23	6427.0833	6	15	as 6 is ✓
✓ 10	10	of their	13	lat	III. ✓
✓ 18	3	G F	40	12	to every ✓
✓ 41	27	in B	57	2	unite; the ✓
✓ 57	25	in the	74	10	on the left at
✓ 78	12	given difference of	87	33	ference of
✓ 101	23	E S E $\frac{1}{4}$ E	120	32	the declina-
✓ 122	13	10000	125	24	latitude and
✓ 125	24	arch; add	125	32	3d example ✓
✓ 127	13	87 deg.	129	21	versed sine ✓

*References omitted.*

page	line	annex
65	8	Plate IV. Fig. 1 and 2. ✓
93	12	Plate II. Fig. 11. ✓
111	26	Plate II. Fig. 13. ✓
114	24	Plate V. Fig. 1. ✓



Plate I.

Fig: 1.

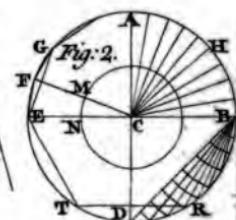
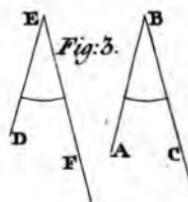
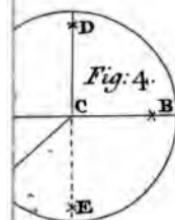
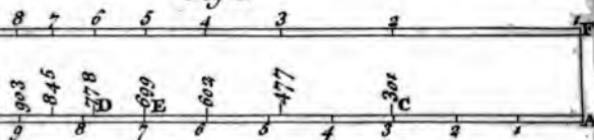


Fig: 7.

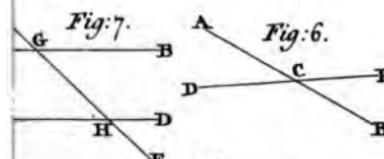


Fig: 5.

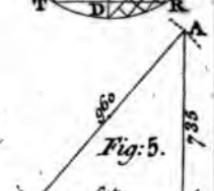


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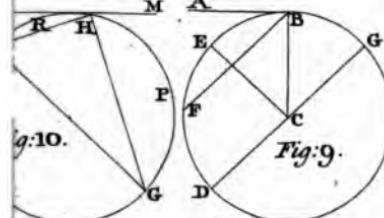


Fig: 8.

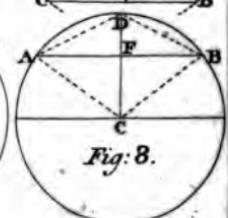


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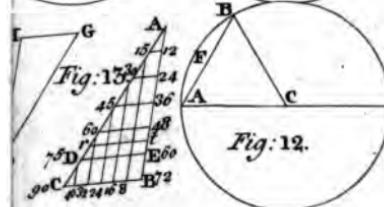


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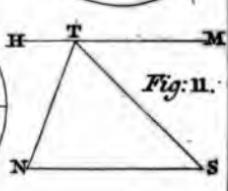


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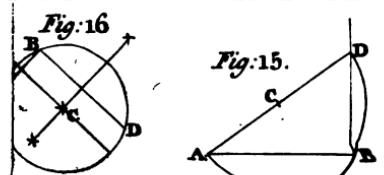


Fig: 14.





## Plate II.

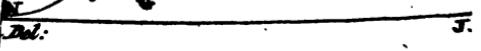
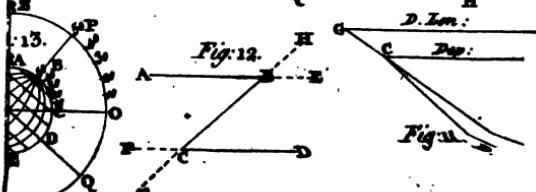
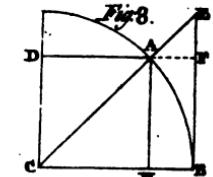
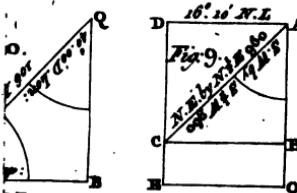
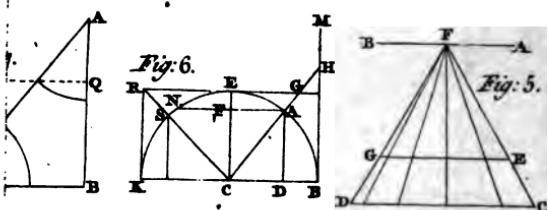
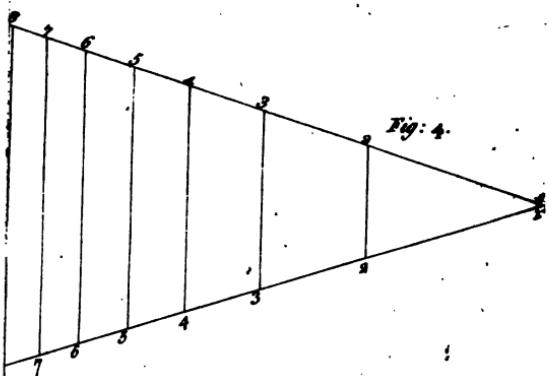
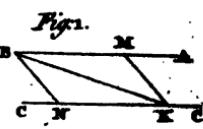
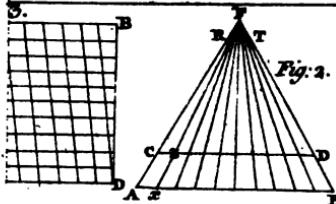




Plate III.

Fig. 2.

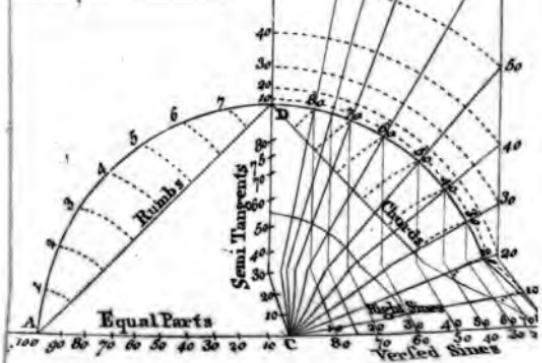
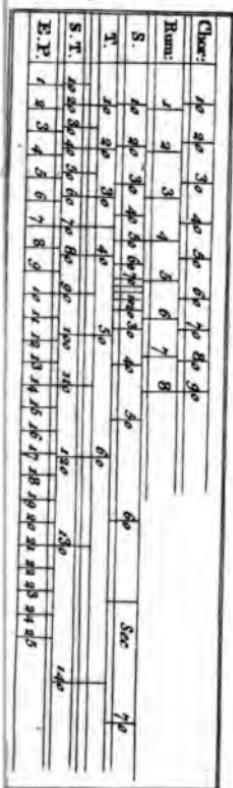
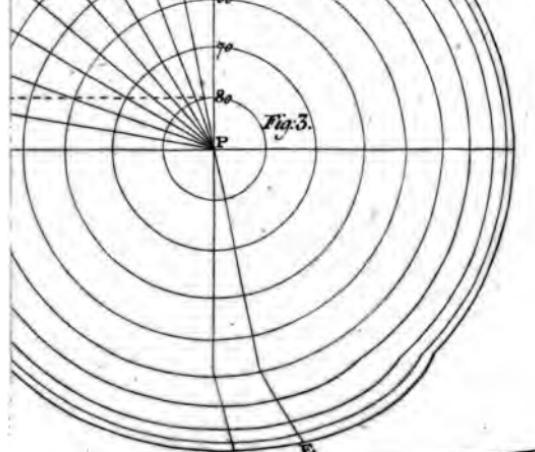
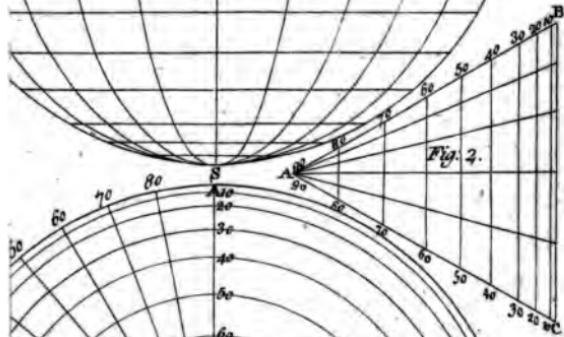
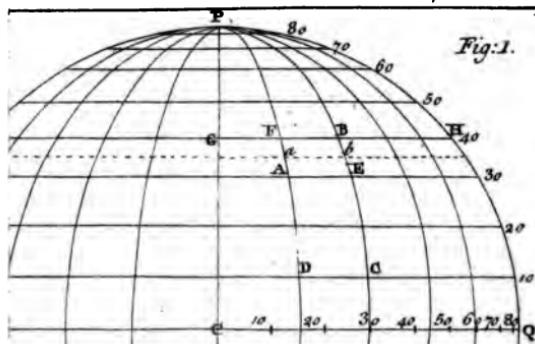


Fig. 1.



Plate IV.



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## Plate V.

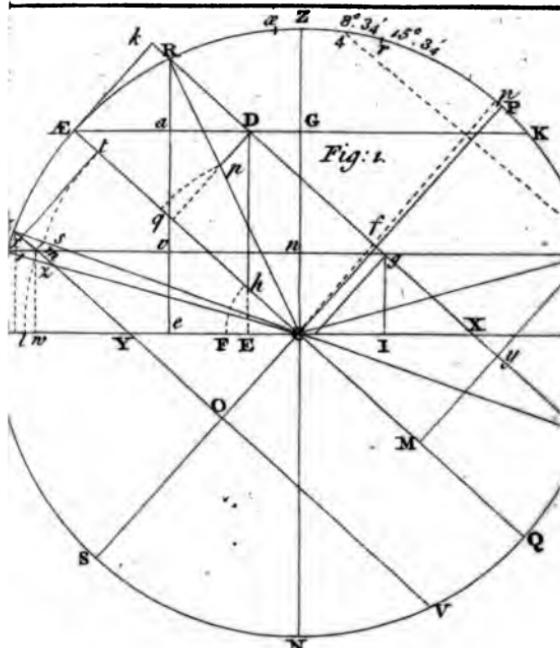
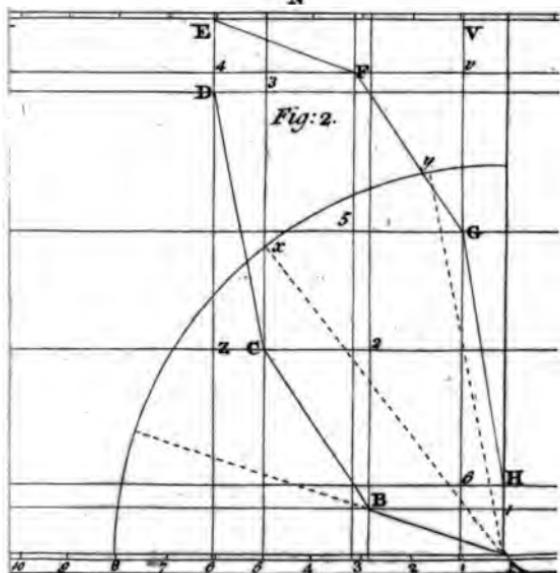


Fig: 2.



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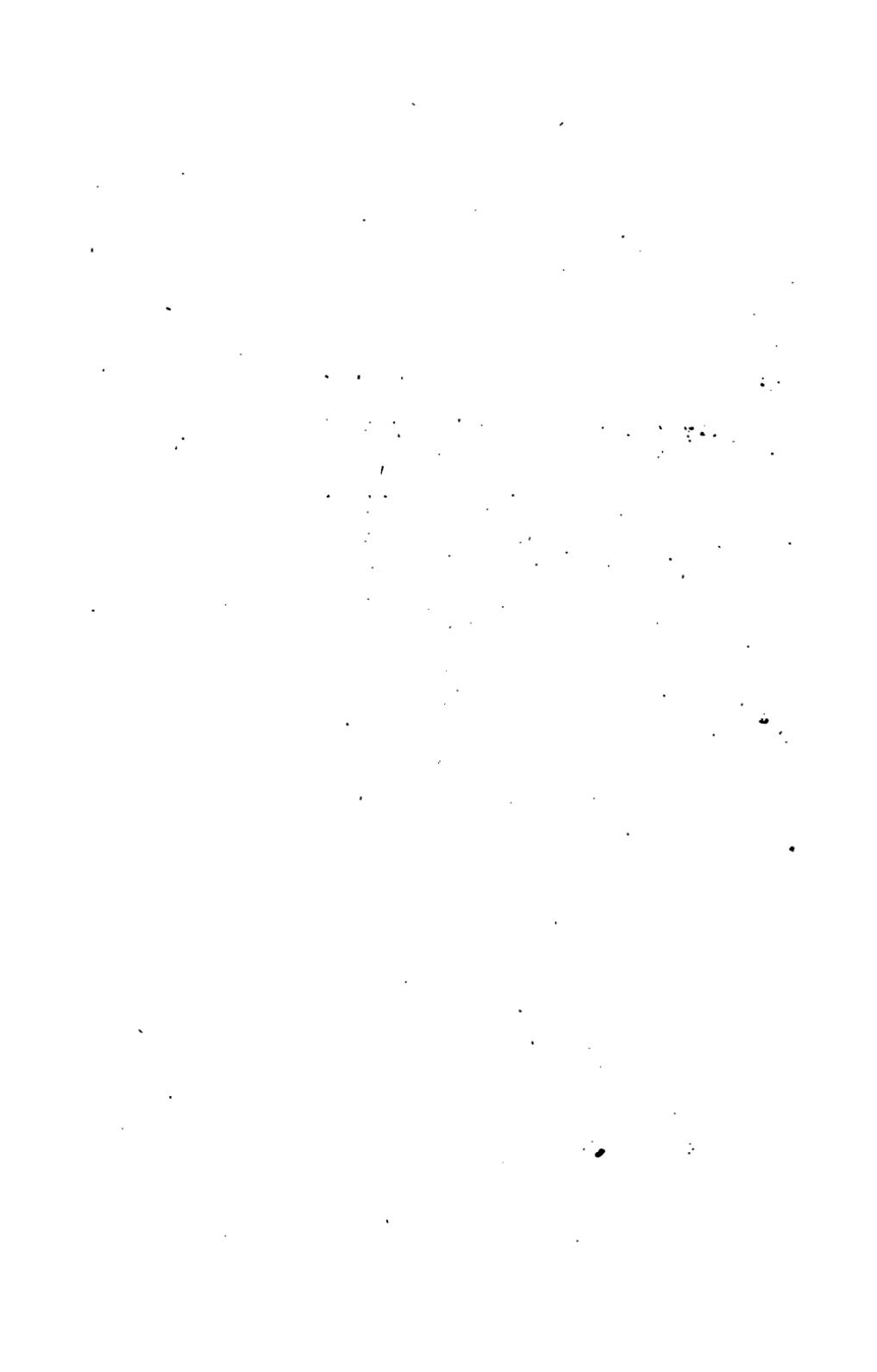


Plate I.

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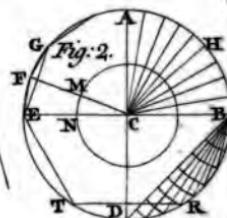
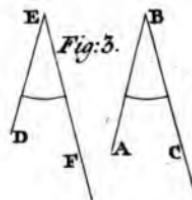
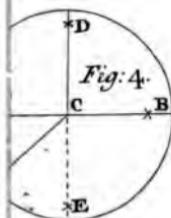
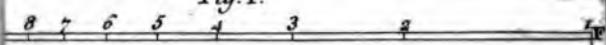


Fig. 7.

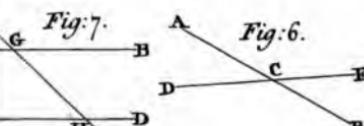


Fig. 5.

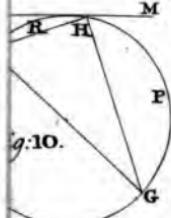


Fig. 9.

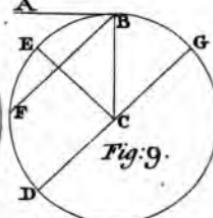


Fig. 8.

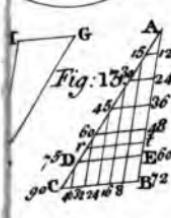


Fig. 12.

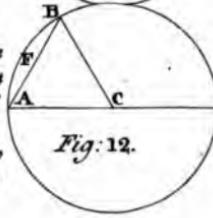


Fig. 11.

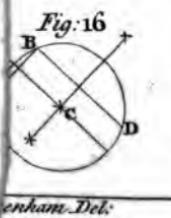


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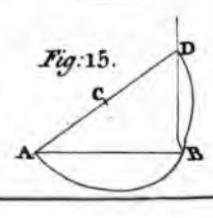


Fig. 14.

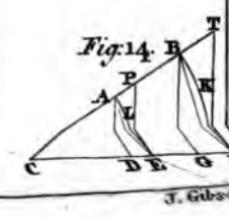




Plate II.

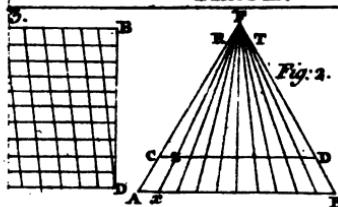


Fig. 2.



Fig. 1.

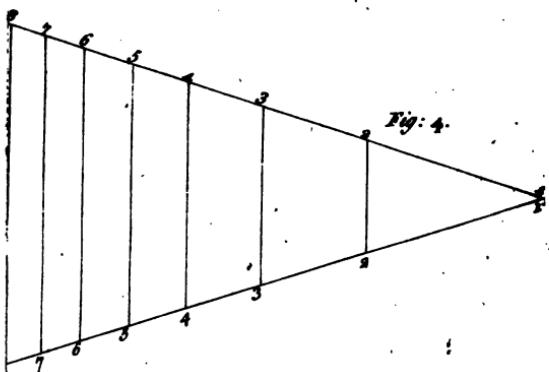


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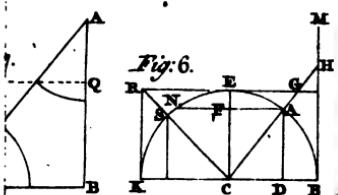


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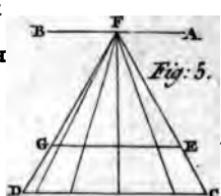


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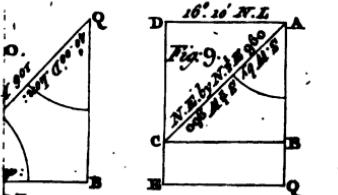


Fig. 9. 16° 16' N.E.

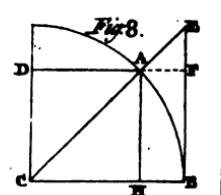


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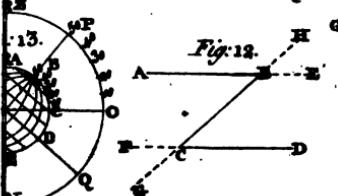


Fig. 12.

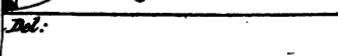


Fig. 11.



Fig. 10.



Fig. 10.



Fig. 10.



Fig. 10.



Fig. 10.



Fig. 10.



Fig. 10.



Fig. 10.

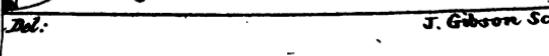


Fig. 10.



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Fig. 10.

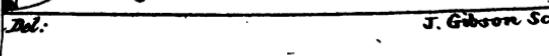


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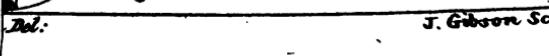


Fig. 10.



Fig. 10.



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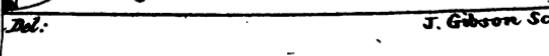


Fig. 10.

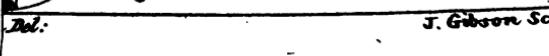


Fig. 10.



Fig. 10.

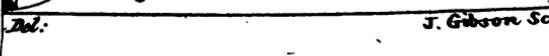


Fig. 10.

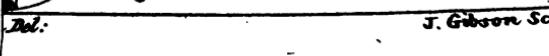


Fig. 10.



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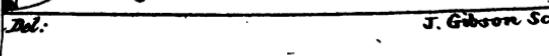


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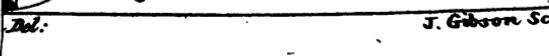


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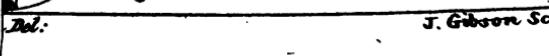


Fig. 10.



Fig. 10.



Fig. 10.



Fig. 10.



Fig. 10.



Fig. 10.





Plate III.

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Fig. 2.

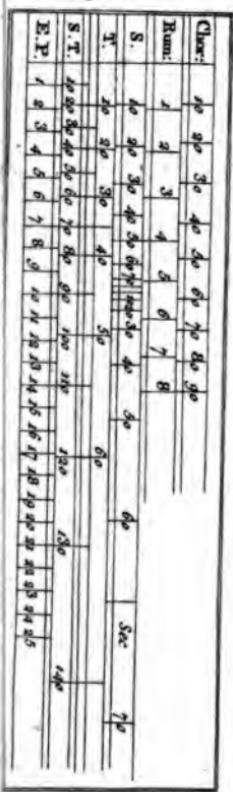


Fig. 1.

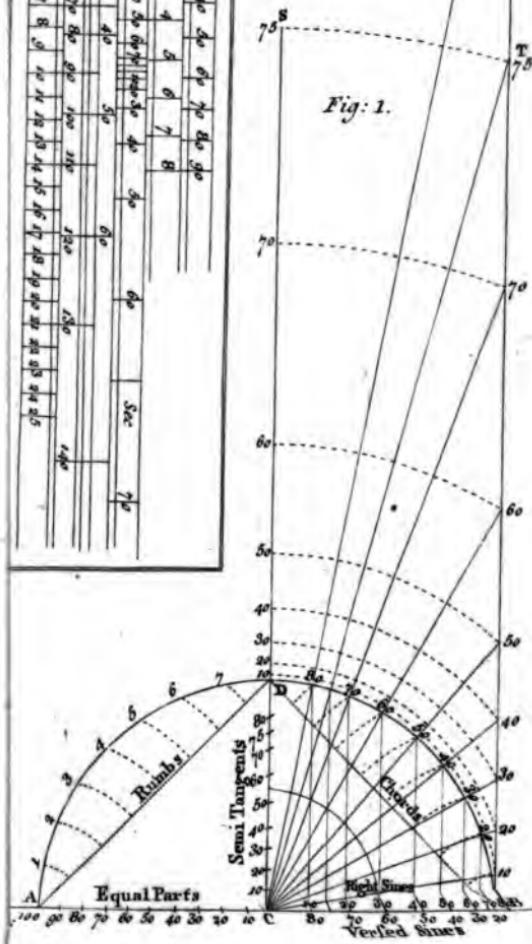
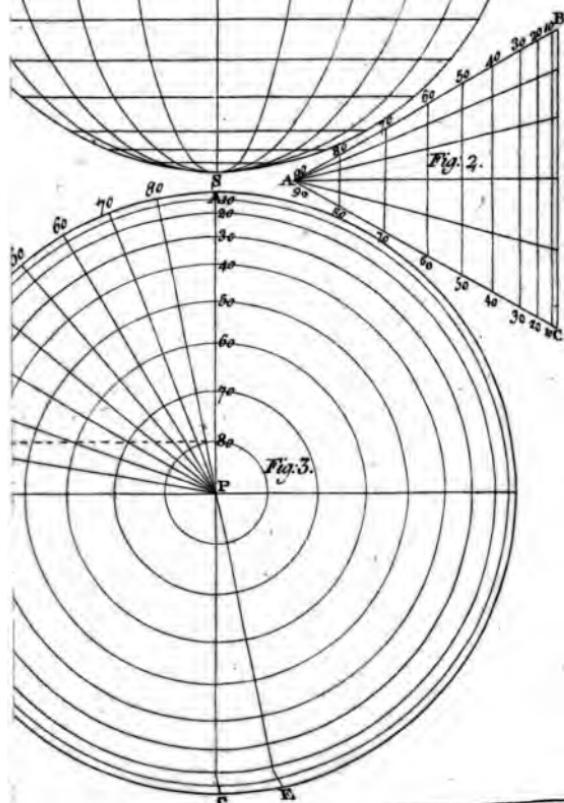
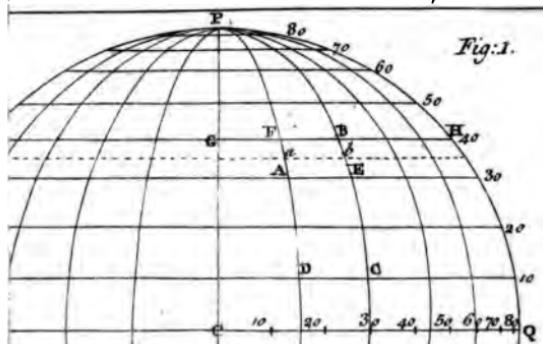




Plate IV.



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Plate V.

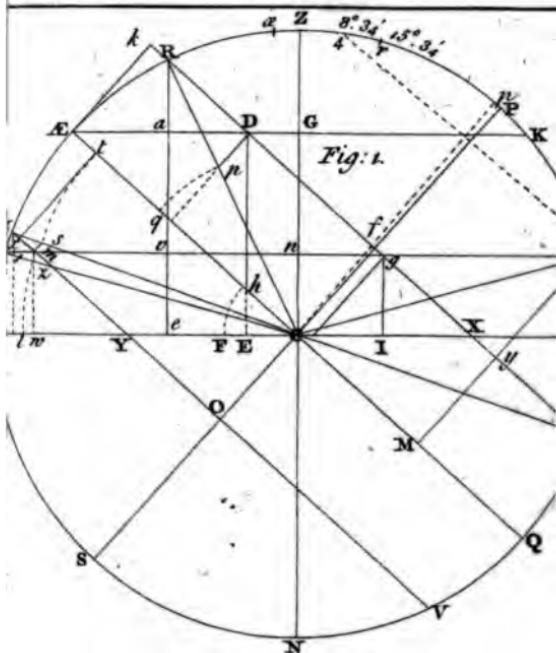
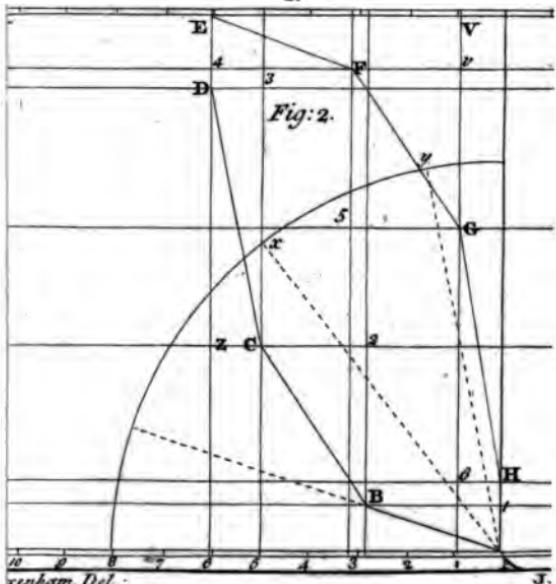


Fig: 1.



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